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$$(\ln x)^2 \ll \sqrt{x}$$

$$\lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x^{1/2}} \frac{\infty}{\infty} \xrightarrow{L'H} \frac{2 \ln x \cdot \frac{1}{x}}{\frac{1}{2} x^{-1/2}}$$

$$= \frac{2 \ln x \cdot \frac{1}{x}}{\frac{1}{2} \cdot \frac{1}{x^{1/2}}} = \frac{4 \ln x \cdot \frac{\infty}{\infty}}{\frac{\infty}{\infty}} \xrightarrow{L'H}$$

$$\frac{\frac{4}{x}}{\frac{1}{2} x^{-1/2}} = \frac{4}{\sqrt{x}} \rightarrow 0$$

$$(\ln x)^4 \ll x^{1/10}$$

$$\frac{(\ln x)^4}{x^{1/10}} \rightarrow \frac{4(\ln x)^3 \cdot \frac{1}{x}}{\frac{1}{10} x^{-9/10}} = 40(\ln x)^3 \frac{x^{9/10}}{x} =$$

$$\frac{4(\ln x)^3}{x^{1/10}} \xrightarrow{LR} \frac{12(\ln x)^2 \cdot \frac{1}{x}}{\frac{1}{10} x^{-9/10}} = \frac{120(\ln x)^2}{x^{1/10}}$$

$$\xrightarrow{LR} \frac{240 \ln x}{\frac{1}{10} x^{1/10}} = \frac{2400 \ln x}{x^{1/10}} \xrightarrow{LR} \frac{2400}{x} \frac{1}{\frac{1}{10} x^{-9/10}}$$

$$= \frac{24,000}{x^{1/10}} \rightarrow 0$$