



## 10.1 examples

**Calculus BC: section 10.1 – Sequences**

Example 1 – Recursively Defined Sequence <sup>FIBONACCI</sup> 1, 1, 2, 3, 5, 8, 13, ...

Compute  $a_2, a_3, a_4$  for the sequence recursively defined by

$$a_1 = 1, \quad a_n = \frac{1}{2} \left( a_{n-1} + \frac{2}{a_{n-1}} \right) \quad a_2 = \frac{1}{2} \left( 1 + \frac{2}{1} \right) = \frac{3}{2}$$

$$a_3 = \frac{1}{2} \left( \frac{3}{2} + \frac{2}{\frac{3}{2}} \right) = \frac{1}{2} \left( \frac{3}{2} + \frac{4}{3} \right) = \frac{17}{12}$$

Example 2 – Proving Convergence of a Sequence

Let  $a_n = \frac{n+4}{n+1}$ . Prove formally that  $\lim_{n \rightarrow \infty} a_n = 1$

(limit is 1)

$$f(x) = \frac{x+4}{x+1}$$

Example 3

Find the limit of the sequence  $0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots \rightarrow$  |

$$a_n = \frac{n}{n+1} \quad n \geq 0$$

Example 4

Calculate  $\lim_{n \rightarrow \infty} a_n$ , where  $a_n = \frac{n + \ln n}{n^2}$   $a_1 = 1$

$$f(x) = \frac{x + \ln x}{x^2} \xrightarrow{L'H} \frac{1 + \frac{1}{x}}{2x} \rightarrow 0$$

$a_2 = \frac{2 + \ln 2}{4}$   
 $a_3 = \frac{3 + \ln 3}{9}$

Example 5 – Limit of Balmer Wavelengths

Calculate the limit of the Balmer wavelengths  $b_n = \frac{364.5n^2}{n^2 - 4}$ , where  $n \geq 3$ .

$$\longrightarrow 364.5$$

Example 6 – Limit of a Geometric Sequence

Prove that 
$$\lim_{n \rightarrow \infty} r^n = \begin{cases} 0 & \text{if } 0 < r < 1 \\ 1 & \text{if } r = 1 \\ \text{diverges to } \infty & \text{if } r > 1 \end{cases}$$

Example 7

Show that if  $\lim_{n \rightarrow \infty} |a_n| = 0$  then  $\lim_{n \rightarrow \infty} a_n = 0$

Example 8

Prove that  $\lim_{n \rightarrow \infty} \frac{R^n}{n!} = 0$  for all R.  $a_1 = R$   
 $a_2 = \frac{R^2}{2!}$

$$a_n = a_{n-1} \cdot \frac{R}{n}$$

$$\lim_{n \rightarrow \infty} \frac{R}{n} = 0 \quad \lim_{n \rightarrow \infty} \sqrt[n]{n} \rightarrow n^{1/n}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n \rightarrow e \quad \lim_{n \rightarrow \infty} \sqrt[n]{n^2} \rightarrow 1^2$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{r}{n}\right)^n \rightarrow e^r \quad \rightarrow 1$$

Example 9

Calculate  $\lim_{n \rightarrow \infty} e^{\frac{3n}{n+1}}$   $\frac{561}{1, 3-6, 11, 14-17}$

Example 10

Verify that  $a_n = \sqrt{n+1} - \sqrt{n}$  is decreasing and bounded below.

Does  $\lim_{n \rightarrow \infty} a_n$  exist?

Verify that  $\{a_n\}$  is decreasing and bounded below.  
Does  $\lim_{n \rightarrow \infty} a_n$  exist?

Example 11

Show that the following sequence is bounded and increasing:

$$a_0 = 0, a_1 = \sqrt{2}, a_2 = \sqrt{2 + \sqrt{2}}, a_3 = \sqrt{2 + \sqrt{2 + \sqrt{2}}}$$

Prove that  $\lim_{n \rightarrow \infty} a_n$  exists and compute its value.