



10.2 examples

Calculus BC, section 10.2 – Summing an Infinite Series

Example 1 – Telescoping Series

Investigate the following series numerically:

$$S = \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{1(2)} + \frac{1}{2(3)} + \frac{1}{3(4)} + \frac{1}{4(5)} + \dots$$

Then compute the sum S using the identity:

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \quad \sum \frac{1}{n(n+1)} = \sum \frac{1}{n} - \sum \frac{1}{n+1}$$

$$S_1 = \frac{1}{1} - \frac{1}{2} = \frac{1}{2}$$

$$S_2 = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} = \frac{2}{3}$$

$$S_3 = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} = \frac{3}{4}$$

$$S_4 = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} = \frac{4}{5}$$

$$S_n = \frac{n}{n+1} = 1 - \frac{1}{n+1}$$

$$\lim_{n \rightarrow \infty} S_n = 1$$

$$n \rightarrow \infty$$

Example 2 – Difference between a Sequence and a Series

Discuss the difference between $\{a_n\}$ and $\sum_{n=0}^{\infty} a_n$, where $a_n = 3^{-n} = \frac{1}{3^n}$

$$\begin{array}{r} 572 \\ \hline 1, 3-6, 9, 11 \end{array}$$

Example 3

Evaluate $\sum_{n=3}^{\infty} 7\left(-\frac{3}{4}\right)^n$

Example 4 – Using the Divergence Test

Does $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1} = -\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{4}{5} - \dots$ converge?

Example 5 – A Divergent Series where $\{a_n\}$ Tends to Zero

Show that $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots$ is divergent.

Example 6

Evaluate $S = \sum_{n=0}^{\infty} \frac{2+3^n}{5^n}$

What is The sum of a
geometric series?

$$S_N = c + cr + cr^2 + cr^3 + \dots + cr^{N-1}$$

$$rS_N = cr + cr^2 + cr^3 + cr^4 + \dots + cr^N$$

$$S_N - rS_N = c - cr^N$$

$$S_N(1-r) = c - cr^N$$

$$S_N = \frac{c(1-r^N)}{1-r}$$

FINITE

$$|r| < 1 \quad \lim_{N \rightarrow \infty} S_N = \frac{c}{1-r} \quad \text{INFINITE}$$