10.3 examples

Thursday, March 14, 2019 9:35 AM

10.3 examples

Calculus BC: Convergence of Series with Positive Terms (section 10.3)

Example 1 – Divergence of the Harmonic Series

Show that $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. $= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$

$$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{R \to \infty} \int_1^{R} \frac{1}{x}\,dx$$

$$= \ln|x| \Big|_1^R = \ln R - \ln 1$$

Example 2

Determine whether $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2}$ converges. $\frac{1}{4} + \frac{2}{25} + \frac{3}{100} + \cdots$

$$\int_1^{\infty} \frac{x}{(x^2+1)^2}\,dx$$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$\frac{1}{2}\int_2^{\infty} \frac{du}{u^2}$ converges

so $\sum$ converges by the Integral Test

Example 3

Show that $S = \sum_{n=1}^{\infty} 2^{-n!}$ converges. $= \frac{1}{2^{1!}} + \frac{1}{2^{2!}} + \frac{1}{2^{3!}} + \frac{1}{2^{4!}}$

compare to $\sum_{n=1}^{\infty} 2^{-n}$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{64} + \frac{1}{2^{24}}$

$\sum 2^{-n!} < \sum 2^{-n}$ for $n > 2$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

converges

$\sum 2^{-n!}$ converges by Comparison Test

Example 4

Show that $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\,3^n}$ converges. $\frac{1}{3} + \frac{1}{\sqrt{2}\cdot 9} + \frac{1}{\sqrt{3}\cdot 27} + \cdots$

$\sum \frac{1}{3^n} > \sum \frac{1}{\sqrt{n}\,3^n}$

for $n > 1$

$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

geometric $r = \frac{1}{3}$

(converges)

$\sum \frac{1}{\sqrt{n}\,3^n}$ converges

by Comparison Test

Example 5

Determine whether $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ converges. $\frac{1}{\ln 2} + \frac{1}{\ln 3} + \frac{1}{\ln 4} + \ldots$

$\sum \frac{1}{n}$ diverges

1.4 0.9 0.7

$\sum \frac{1}{\ln n} > \sum \frac{1}{n}$

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$

$\sum \frac{1}{\ln n}$ diverges by Comparison Test

581: 19, 20, 24, 25, 27, 29

Example 6 – Using the Comparison Test Correctly

Study the convergence of $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$.

Example 7

Show that $\sum_{n=2}^{\infty} \frac{n^2}{n^4 - n}$ converges.

$\frac{1}{4} + \frac{1}{9} + \frac{1}{16}$

$\frac{4}{16-2} + \frac{9}{81-3} + \frac{16}{256-4}$

Try Comparison Test $\quad \frac{n^2}{n^4-n} > \frac{1}{n^2}$

Try Limit Comparison Test

$$\lim_{n\to\infty} \frac{\frac{1}{n^2}}{\frac{n^2}{n^4-n}} = \frac{n^4-n}{n^4} \to 1$$

$1 - \frac{1}{n^3}$

$\sum \frac{n^2}{n^4-n}$ using limit comp

Example 8

Determine whether $\sum_{n=3}^{\infty} \frac{1}{\sqrt{n^2-4}}$ converges.

$$\frac{1}{\sqrt{n^2-4}} > \frac{1}{n}$$

$\uparrow$

diverges

$$\sum \frac{1}{\sqrt{n^2+4}}$$

$$\frac{\frac{1}{n}}{\frac{1}{\sqrt{n^2+4}}} = \frac{\sqrt{n^2+4}}{n} \longrightarrow 1$$

$\sum \frac{1}{\sqrt{n^2+4}}$ diverges by L.C.T.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$ diverges

$$\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$$ converges

581; 3, 6, 7, 9, 10, 13