10.4 examples

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Calculus BC: Absolute and Conditional Convergence (10.4)

$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

<u>Example 1</u>

Verify the convergence of S = $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$

converges, by the Leibniz Test

$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} \ldots$ converges absolutely

Example 2

Does $S = \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ converge absolutely?

Example 3

Show that $S = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ is conditionally convergent and that $0 \le S \le 1$.

$S_1 = \frac{1}{\sqrt{1}} = 1$

$S_2 = 1 - \frac{1}{\sqrt{2}} = 0.3$

$S_3 = S_2 + \frac{1}{\sqrt{3}} = 0.9$

$S_4 = S_3 - \frac{1}{2} = 0.4$

$S_2 = 0.3$

$a_3 = \frac{1}{\sqrt{3}} = 0.57$

Example 4 – The Alternating Harmonic Series

Show that $S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges conditionally. Then

(a) Show that $|S_6 - S| \leq \frac{1}{7}$ $\quad a_7 = \frac{1}{7} \quad$ error $< \frac{1}{7}$

(b) Find an N such that S_N approximates S with an error less than 10^{-3}

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} \cdots - + \frac{1}{999} + \frac{1}{1000}$$

$$a_{1000} = \frac{1}{1000} \qquad \text{error} \leq \frac{1}{1001}$$

Conditionally convergent

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots$$

alternating harmonic series

$$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} - \cdots$$

581: 67, 70

588: 3-7, 10-12

$$\sum_{n=1} (-1)^{n+1} \cdot \frac{1}{n^2} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots$$

How many terms needed so that error $< 10^{-6}$?

error $< \frac{1}{1000^2}$

$\frac{1}{n^2} < 10^{-6}$ $\quad n^2 > 10^6$

last included term $\frac{1}{1000^2}$

first omitted $\frac{1}{1001^2}$

$$\sum_{n=1} (-1)^{(n+1)} \cdot \frac{1}{n^2}$$

$N = ?$ for error $< 10^{-7}$?

$$\frac{1}{n^2} < \frac{1}{10^7} \qquad \text{error} < \frac{1}{3163^2}$$

$$n^2 > 10^7$$

$$N = 3162$$