10.5 examples

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10.5 examples

Calculus BC: The Ratio and Root Tests (section 10.5)

Example 1 $= \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$

Prove that $S = \sum_{n=1}^{\infty} \frac{1}{n!}$ converges.

$a_n = \frac{1}{n!}$ $\qquad$ $\frac{\frac{1}{(n+1)!}}{\frac{1}{n!}}$

$a_{n+1} = \frac{1}{(n+1)!}$

$\longrightarrow \frac{n!}{(n+1)!} = \frac{1}{n+1} \longrightarrow 0$

$\sum$ converges by ratio test

5 & 8: 13, 14, 15, 24, 29

Example 2

Apply the Ratio Test to determine if $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ converges. $\frac{1}{2}$ $\frac{4}{4}$ $\frac{9}{8}$ $\frac{16}{16}$

$$\frac{\frac{(n+1)^2}{2^{n+1}}}{\frac{n^2}{2^n}} = \frac{(n+1)^2 \cdot 2^n}{n^2 \, 2^{n+1}} \qquad \frac{(n+1)^2}{n^2} = \left(\frac{n+1}{n}\right)^2$$

$$= \left(1 + \frac{1}{n}\right)^2 \to 1$$

$$\to \frac{1}{2}$$

$\sum$ converges by Ratio Test

Example 3

Determine if $\sum_{n=1}^{\infty}(-1)^n \frac{n!}{100^n}$ converges.

$$\frac{\frac{(n+1)!}{100^{n+1}}}{\frac{n!}{100^n}} = \frac{100^n\,(n+1)!}{100^{n+1}\,n!}$$

$$\frac{n+1}{100} \longrightarrow \infty$$

$\sum$ diverges

Example 4 – Ratio Test Inconclusive

Show that $\rho = 1$ for both $\sum_{n=1}^{\infty} n^2$ and $\sum_{n=1}^{\infty} n^{-2}$.

Conclude that the Ratio Test is inconclusive when $\rho = 1$.

Example 5

Determine whether $\sum_{n=1}^{\infty}\left(\frac{n}{2n+3}\right)^n$ converges.

$\sqrt[n]{\left(\frac{n}{2n+3}\right)^n} = \frac{n}{2n+3} \longrightarrow \frac{1}{2}$

$\sum$ converges

(19) $1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{5} + \frac{1}{3} - \frac{1}{7} + \frac{1}{4} - \frac{1}{9} + \frac{1}{5} - \frac{1}{11} \cdots$

$$\sum_{n=1} \frac{1}{n} - \frac{1}{(2n+1)}$$

$$\sum \frac{(2n+1) - n}{n(2n+1)}$$

$$\sum \frac{n+1}{n(2n+1)}$$

LCT $\dfrac{\frac{1}{n}}{\frac{n+1}{n(2n+1)}}$

$$= \frac{n(2n+1)}{n(n+1)} \longrightarrow \frac{2n+1}{n+1} \longrightarrow 2$$

$\sum$ diverges

(13) $S = \sum_{n=1} (-1)^{n+1} \cdot \frac{1}{n^3}$

S_n $\qquad S_{10} = 0.9011$

$0.9 \leq S \leq 0.902$

$\text{error} \leq \frac{1}{11^3} = 0.00075$

$0.9011 - 0.00075 \leq S \leq 0.9011 + 0.00075$

$$\frac{dy}{dx} = 2x - y \qquad \frac{dy}{dx} = m$$

$$m = 2x - y = 2x - (mx + b)$$

$$m = 2x - mx - b$$

$$m + b = x(2 - m)$$

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$$\lim_{n \to \infty} \sqrt[n]{n} \to 1$$

$$\sqrt[n]{n^2} \longrightarrow 1^2 = 1$$

$$\sqrt[n]{n^n} \longrightarrow n$$

$$\sum_{n=1}^{\infty} (-1)^n n e^{-n}$$

Root Test

$$\sqrt[n]{n e^{-n}} = e^{-1} < 1$$

✓

Leibniz Test

$a_n \to 0$

$a_n > a_{n+1}$

$$\frac{n}{e^n} \to 0$$

592: 2, 10, 13, 14,
34, 35, 36, 38