10.6 examples

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Calculus BC – Power Series (section 10.6)

Example 1 – Using the Ratio Test

For which values of x does $F(x) = \sum_{n=0}^{\infty} \frac{x^n}{2^n}$ converge?

$\sqrt[n]{\left|\frac{x^n}{2^n}\right|}$

check endpoints.

$1 + \frac{-2}{2} + \frac{4}{4} + \cdots$

$\left|\frac{x}{2}\right| < 1$

$(-2, 2)$ is interval of convergence

Example 2

Determine the convergence of F(x) = $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}(x-5)^n$

$$\left|\frac{(x-5)^n}{n}\right| \xrightarrow{\text{Root}} \left|\frac{x-5}{1}\right| < 1$$

$(4, 6]$ is interval of convergence

check endpts: $-\frac{(4-5)^1}{1} + \frac{(4-5)^2}{2} - \frac{(4-5)^3}{3}$

$-\frac{(6-5)^1}{1} + \frac{(6-5)^2}{2} - \frac{(6-5)^3}{3}$

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Example 3 – Infinite Radius of Convergence

Show that $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges for all x

Example 4 – Using the Formula for Geometric Series

Prove that $\frac{1}{1-2x} = \sum_{n=0}^{\infty} 2^n x^n$ for $|x| < \frac{1}{2}$

$(2x)^n$ $\quad$ $r = 2x$ $\quad$ $|2x| < 1$

sum $= \frac{1}{1-2x}$

Example 5

Prove that $\frac{1}{2+x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{n+1}}$.

For which x is this formula valid?

Example 6 – Differentiating a Power Series

$$\sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}$$

Prove that $\dfrac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \ldots$ for -1 < x < 1

$$\frac{1}{1-x} = (1-x)^{-1} \qquad \frac{d}{dx} = -(1-x)^{-2}(-1)$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 \qquad = \frac{1}{(1-x)^2}$$

$$\sum x^n$$

Example 7 – The Power Series for f(x) = $\tan^{-1}$ x via Integration

Prove that for -1 < x < 1,

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$

$$\int \frac{1}{1+x^2} = \int 1 - x^2 + x^4 - x^6 + x^8 - \ldots$$

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$$1 + x + x^2 + x^3 + \cdots\cdots = \frac{1}{1-x}$$

$$(\text{if } |x| < 1)$$

$$1 - x^2 + x^4 - x^6 + \cdots = \frac{1}{1 - {-x^2}} = \frac{1}{1+x^2}$$

Find an infinite series that matches

$$g(x) = \frac{1}{5+x} = \frac{1}{5(1+\frac{x}{5})} = \frac{\frac{1}{5}}{1+x/5}$$

$$r = -\frac{x}{5}$$

$$\frac{1}{5} - \frac{x}{25} + \frac{x^2}{125} - \frac{x^3}{625} + \cdots$$

$$g(x) = \frac{1}{1+x} = 1 - x + x^2 - x^3 \cdots$$

$$\int g(x)\,dx = \int \cdots -$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots$$

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