11.1 examples

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Calculus BC: Parametric Equations (section 11.1)

Example 1 – Eliminating the Parameter

Describe the parametric curve $c(t) = (2t - 4, 3 + t^2)$ in the form $y = f(x)$.

Example 2

A bullet follows the trajectory (t in seconds, distance in feet)

$X(t) = 200t \qquad y(t) = 400t - 16t^2 \qquad 0 \le t \le 25$

Find the bullet's height at t = 5 and its maximum height.

$\frac{dy}{dt} = 400 - 32t = 0 \qquad t = 12.5 \text{ s}$

$y(12.5) = 2500'$

Example 3 – Parametric Representation of a Line

Show that the line through the point P = (a, b) with slope m has parametrization c(t) = (a + t, b + mt), with $-\infty < t < \infty$.

Example 4 – Parametrization of an Ellipse

Show that the ellipse $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$ is parametrized by

c(t) = (a cos t, b sin t), $0 \le t \le 2\pi$.

Graph the case a = 4, b = 2, and indicate the points corresponding to

t = 0, $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$.

Example 5 – Parametrizations and Paths versus Curves

Describe the motion of a particle moving along the paths (for $-\infty < t < \infty$):

a) $c_1(t) = (t^3, t^6)$ b) $c_2(t) = (t^2, t^4)$ c) $c_3(t) = (\cos t, \cos^2 t)$

Example 6 – Using Symmetry to Sketch a Loop

Sketch the curve parametrized by $c(t) = (t^2 + 1, t^3 - 4t)$

Example 8

Let $c(t) = (t^2 + 1, t^3 - 4t)$. Find the equation of the tangent line at $t = 3$ and find the points where the tangent is horizontal.

$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = 3t^2 - 4 \qquad \frac{dy}{dx} = \frac{3t^2 - 4}{2t} = 0$

$= \frac{3(9) - 4}{6} = \frac{23}{6} \qquad c(3) = (10, 15)$

$y - 15 = \frac{23}{6}(x - 10)$

$3t^2 - 4 = 0$

$t = \pm\sqrt{4/3}$

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