11.2 examples

Calculus AB: Arc Length and Speed (section 11.2)

<u>Example 1</u> – Length of a Circular Arc

Calculate the length s of the arc $0 \le \theta \le \theta_0$ of a circle of radius R.

Example 2 – Length of the Cycloid

Calculate the length s of one arch of the cycloid generated by a circle of radius R = 2 (Figure 3).

$x = 2(t - \sin t) \qquad y = 2(1 - \cos t)$

Example 3

A particle travels along the path $c(t) = (2t, 1 + t^{3/2})$ (t in minutes, distance ir feet).

a) Find the speed at $t = 1$.

b) Compute distance traveled s and displacement d during the first 4 minutes.

$x' = 2 \qquad y' = \frac{3}{2} t^{1/2}$

$(x')^2 = 4 \qquad (y')^2 = \frac{9}{4} t$

speed $= \sqrt{4 + \frac{9}{4} t}$

speed $(1) = \sqrt{4 + \frac{9}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2}$

distance travelled $= \int_0^4 \sqrt{4 + \frac{9}{4} t}\, dt = 11.517'$

displacement: initial position $= (0, 1)$

ending position $= (8, 9) \qquad \sqrt{64 + 64} = 11.313$

Example 4 – Angular Velocity

Let $c(t) = (R \cos \omega t, R \sin \omega t)$ be a circular path, where ω is a constant (called the angular velocity, in units of radians per unit time). Show that $c(t)$ is a path of constant speed. Find the angular velocity ω of a counterclockwise path if $R = 3$ m and the speed is 12 m/s.

Example 5 – Surface Area

Use the formula for surface area to find the surface area of the cone generated by revolving c(t) = (t, mt) about the x-axis for $0 \le t \le A$.

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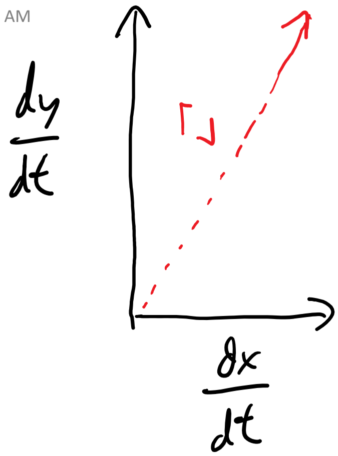

speed?

$$\text{speed} = \|\vec{v}\| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

arc length —

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$D = RT$

velocity vector = $\langle t^2, \sin t \rangle$

position vector = $\langle \cos t, e^t \rangle$

Find speed if $c(t) = (3\sin 5t, 8\cos 5t)$
at $t = \pi/4$

Find distance travelled on $(0, \pi/4)$

Find displacement on $(0, \pi/4$

$x' = 15\cos 5t \qquad y' = -40\sin 5t$

$(x')^2 = 225\cos^2 5t \qquad (y')^2 = 1600\sin^2 5t$

$\text{speed} = \sqrt{225\cos^2 5t + 1600\sin^2 5t} =$

$\text{speed}(\pi/4) = 30.207$

distance = 21.522

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