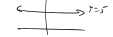


2.2 examples

Calculus AB, section 2.2

Example 1

$\lim_{x \rightarrow 7} 5 = 5$ $\lim_{x \rightarrow 4} (3x+1) = 13$
 a. $x \rightarrow 7$ b. $x \rightarrow 4$



Example 2

Investigate graphically and numerically $\lim_{x \rightarrow 9} \frac{x-9}{\sqrt{x}-3} = \frac{0}{0}$

x	$\frac{x-9}{\sqrt{x}-3}$	x	$\frac{x-9}{\sqrt{x}-3}$
8.9		9.1	
8.99		9.01	
8.999		9.001	
8.9999		9.0001	

= 6

Example 3

Investigate graphically and numerically $\lim_{x \rightarrow 4} x^2$

x	x^2	x	x^2
3.9		4.1	
3.99		4.01	
3.999		4.001	
3.9999		4.0001	

Limit = function value when function is continuous

Example 4

Verify numerically that $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$

(if you use a different base than e, the limit $\neq 1$)

$h \rightarrow 0^+$	$\frac{e^h - 1}{h}$	$h \rightarrow 0^-$	$\frac{e^h - 1}{h}$
0.02	1.0101	-0.02	0.990
0.005	1.00250	-0.005	0.99750
0.001	1.000500	-0.001	0.999500
0.0001	1.00005000	-0.0001	0.99995000

Example 5

Investigate graphically and numerically $\lim_{x \rightarrow 0} \sin \frac{1}{x}$

x	$\sin \frac{1}{x}$	x	$\sin \frac{1}{x}$
0.1	-0.5	-0.1	
0.05		-0.05	
0.01	-0.5	-0.01	
0.001		-0.001	
0.0005	0.9	-0.0005	
0.00002		-0.00002	
0.00001	0.04	-0.00001	

The limit does not exist

7, 67, 7,		
7, 23, 24		

Example 6

Investigate the one-sided limits of $f(x) = \frac{x}{|x|}$ as $x \rightarrow 0$. Does the (two-sided) limit exist? [graph is below]



FIGURE 6 Graph of $y = \frac{x}{|x|}$.

sided) limit exist? $\sim \text{no}$

Example 7

The function $f(x)$ shown in Figure 7 is not defined at $c = 0, 2, 4$. Do the one- or two-sided limits exist at these points? If so, find their values.

$\lim_{x \rightarrow 0^+} f = 2$
 $\lim_{x \rightarrow 0^-} f = \text{DNE}$
 $\lim_{x \rightarrow 0} f = \text{DNE}$

$\lim_{x \rightarrow 2^+} f = 1$
 $\lim_{x \rightarrow 2^-} f = 3$
 $\lim_{x \rightarrow 2} f = \text{DNE}$

$\lim_{x \rightarrow 4^+} f = 2$
 $\lim_{x \rightarrow 4^-} f = 2$
 $\lim_{x \rightarrow 4} f = 2$

FIGURE 7

Example 8

Investigate the one-sided limits: [graphs below]

a. $\lim_{x \rightarrow 2^\pm} \frac{1}{x-2}$

b. $\lim_{x \rightarrow 0^\pm} \frac{1}{x^2}$

c. $\lim_{x \rightarrow 0^+} \ln x$

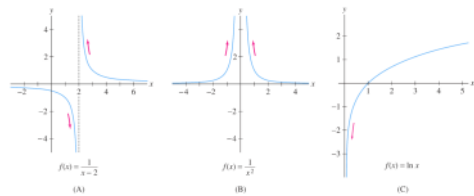
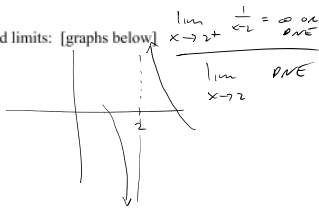


FIGURE 8

Sketch a graph $g(x)$ with

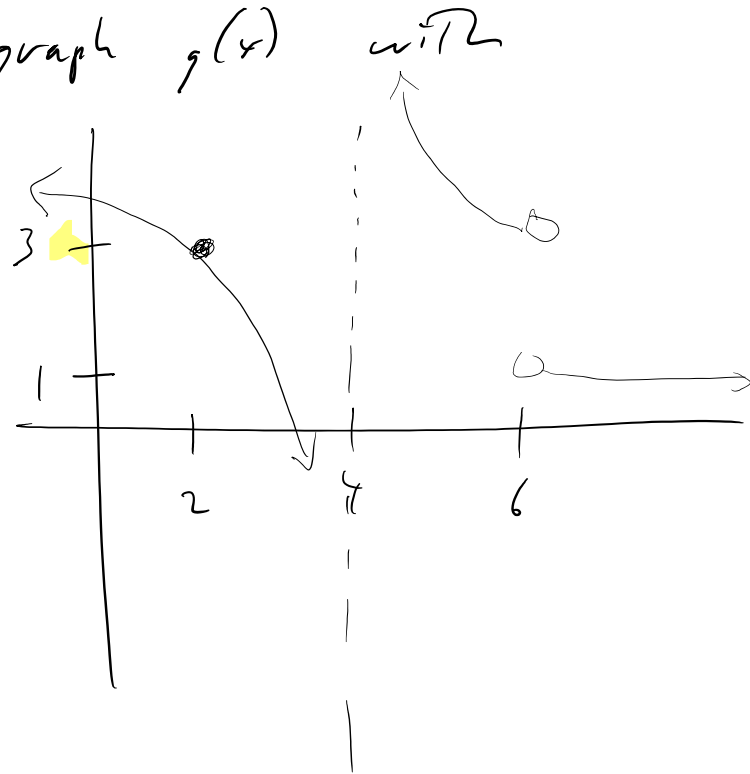
$$\lim_{x \rightarrow 2} g(x) = 3$$

$$\lim_{x \rightarrow 4^-} g(x) = -\infty$$

$$\lim_{x \rightarrow 4^+} g = \infty$$

$$\lim_{x \rightarrow 6^-} g = 3$$

$$\lim_{x \rightarrow 6^+} g = 1$$



76: 5, 8, 31, 34, 37, 38, 47, 49