

## 2.6 examples

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### 2.6 examples

#### Calculus AB, 2.6 Examples

f. 97: 8, 9, 10, 13, 16, 22

#### Example 1

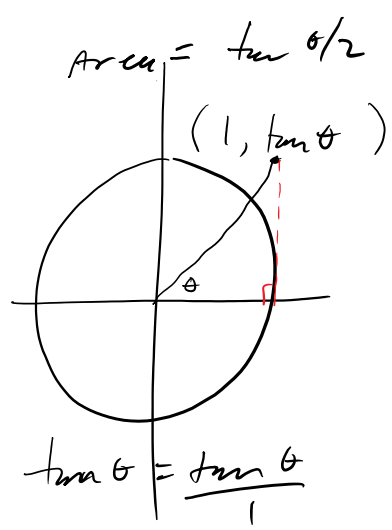
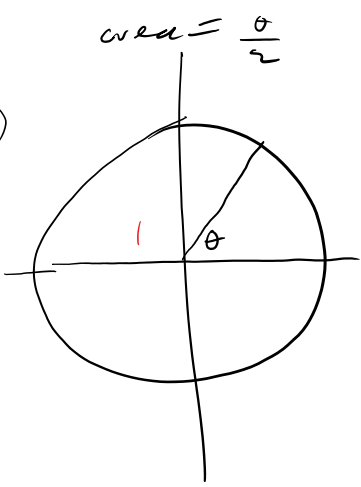
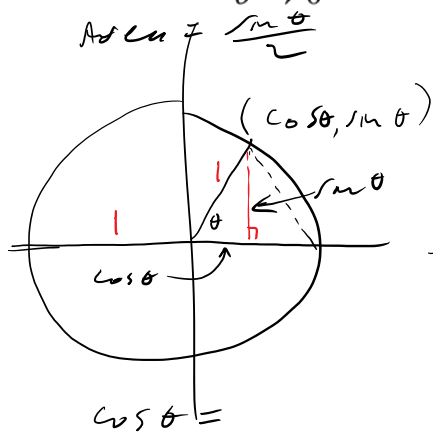
Show that  $\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$   $-1 \leq \sin \frac{1}{x} \leq 1$

$$\lim_{x \rightarrow 0} -x \leq x \sin \frac{1}{x} \leq x$$

$\downarrow$                        $\downarrow$                        $\downarrow$   
0                                      0                                      0

by Squeeze Theorem

Prove Theorem 2:  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$



$$\frac{\tan^2 \theta}{2} \geq \frac{\theta}{2} \geq \frac{\sin^2 \theta}{2}$$

$$\tan \theta \geq \theta \geq \sin \theta$$

$$1 \geq \frac{\sin \theta}{\theta}$$

$$\frac{\sin \theta}{\cos \theta} \geq \theta$$

$$\frac{\sin \theta}{\theta} \geq \cos \theta$$

$$\lim_{\theta \rightarrow 0} \left( \cos \theta \leq \frac{\sin \theta}{\theta} \leq 1 \right) \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \rightarrow 1$$

To prove Theorem 2, first prove Theorem 3:

$$\cos \theta \leq \frac{\sin \theta}{\theta} \leq 1$$

Example 2

Investigate  $\lim_{h \rightarrow 0} \frac{\sin 4h}{h}$  numerically and then evaluate it exactly.