3.10 examples

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3.10 examples

Calculus AB, section 3.10 –
Derivatives of Exponential and Logarithmic Functions

$\frac{d}{dx} b^u = b^u \cdot \ln b \cdot \frac{du}{dx}$

Example 1
Find the derivative of:

a) $y = 4^{3x}$

$\frac{dy}{dx} = 4^{3x} \cdot \ln 4 \cdot 3$

b) $f(x) = 5^{x^2}$

$f'(x) = \ln 5 \cdot 5^{x^2} \cdot 2x$

$\frac{d}{dx} \ln u = \frac{1}{u} \cdot \frac{du}{dx}$

Example 2

Find the derivative of:

a) $y = x \ln x$ $\quad \frac{dy}{dx} = \frac{1}{x} \cdot x + \ln x = 1 + \ln x$

b) $y = (\ln x)^2$ $\quad \frac{dy}{dx} = 2 \ln x \cdot \frac{1}{x} = \frac{2 \ln x}{x}$

Example 3

Find the derivative of:

a) $y = \ln(x^3 + 1)$ $\quad y' = \frac{1}{x^3 + 1} \cdot 3x^2$

b) $y = \ln \sqrt{\sin x}$ $\quad = \ln \left((\sin x)^{1/2} \right)$

$\frac{dy}{dx} = \frac{1}{\sqrt{\sin x}} \cdot \frac{1}{2} (\sin x)^{-1/2} \cdot \cos x$

$\frac{1}{\sqrt{\sin x}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\sin x}} \cdot \cos x =$

$\frac{1}{2} \cot x$

Example 4 – Derivative of the Logarithm to another base

$$\frac{d}{dx}\log_{10} x = \frac{d}{dx}\left(\frac{\ln x}{\ln 10}\right) \qquad \log_a b = \frac{\log_c b}{\log_c a}$$

$$= \frac{1}{\ln 10}\cdot\frac{d}{dx}(\ln x) \qquad \frac{d}{dx}(\ln 10) = 0$$

$$= \frac{1}{\ln 10 \cdot x}$$

$$\frac{d}{dx}\left(\frac{\ln x}{\ln 10}\right) = \frac{\ln 10 \cdot \frac{1}{x} - \ln x \cdot 0}{(\ln 10)^2} = \frac{\frac{1}{x}}{\ln 10}$$

$$= \frac{1}{x \ln 10}$$

Example 5 – Logarithmic Differentiation

Find the derivative of $f(x) = \dfrac{(x+1)^2(2x^2-3)}{\sqrt{x^2+1}}$

Example 6

Find the derivative of: logarithmic differentiation

a) $f(x) = x^x$

$y = x^x$

$\ln y = \ln x^x$

$\frac{d}{dx}(\ln y = x \ln x)$

$\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$

b) $g(x) = x^{\sin x}$

$\frac{dy}{dx} = y(\ln x + 1)$

$\frac{dy}{dx} = x^x(\ln x + 1)$

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$$f(x) = x^{\sin x}$$

$$y = x^{\sin x}$$

$$\frac{d}{dx}\left(\ln y = \ln x^{\sin x} = \sin x \ln x\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\sin x}{x} + \cos x \ln x$$

$$\frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x \ln x\right)$$

$$x^2$$