



## Calculus AB, section 3.10 – Derivatives of Exponential and Logarithmic Functions

### Example 1

Find the derivative of:

$$\text{a) } y = 4^{3x} \quad \frac{dy}{dx} = 4^{3x} \cdot 3 \cdot \ln 4$$

$$\frac{d}{dx} e^x = e^x \ln e$$

$$\text{b) } f(x) = 5^{x^2} \quad \frac{dy}{dx} = 2x \cdot 5^{x^2} \cdot \ln 5$$

Example 2

Find the derivative of:

a)  $y = x \ln x$       $\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x \cdot 1 = 1 + \ln x$

b)  $y = (\ln x)^2$       $\frac{dy}{dx} = 2 \ln x \cdot \frac{1}{x}$

### Example 3

Find the derivative of:

$$\text{a) } y = \ln(x^3 + 1) \quad y' = \frac{1}{x^3 + 1} \cdot 3x^2$$

$$\begin{aligned} \text{b) } y &= \ln \sqrt{\sin x} = \ln \left[ (\sin x)^{1/2} \right] \\ y' &= \frac{1}{\sqrt{\sin x}} \cdot \frac{1}{2} (\sin x)^{-1/2} \cdot \cos x \\ &= \frac{1}{2} \cot x \end{aligned}$$

Example 4 – Derivative of the Logarithm to another base

$$\begin{aligned}\frac{d}{dx} \log_{10} x &= \frac{d}{dx} \left( \frac{\ln x}{\ln 10} \right) = \frac{1}{\ln 10} \frac{d}{dx} \ln x \\ &= \frac{1}{\ln 10} \cdot \frac{1}{x}\end{aligned}$$

Example 5 – Logarithmic Differentiation

Find the derivative of  $f(x) = \frac{(x+1)^2(2x^2-3)}{\sqrt{x^2+1}}$

197; 3, 4, 7, 8, 10, 15, 18, 24

### Example 6

Find the derivative of:

logarithmic differentiation

a)  $f(x) = x^x$  ①  $x = e^{\ln x}$   $f(x) = (e^{\ln x})^x$   
 $f(x) = e^{x \ln x}$   $f'(x) = e^{x \ln x} \cdot (\ln x + \frac{1}{x} \cdot x)$   
 $= e^{x \ln x} (\ln x + 1) = (e^{\ln x})^x (\ln x + 1) = x^x (\ln x + 1)$

②  $y = x^x$   $\ln y = \ln x^x$   $\frac{d}{dx} (\ln y = x \ln x)$   
 $\frac{1}{y} \frac{dy}{dx} = 1 \cdot \ln x + \frac{1}{x} \cdot x = \ln x + 1$   $\frac{dy}{dx} = y (\ln x + 1)$

b)  $g(x) = x^{\sin x}$   $\ln y = \ln x^{\sin x}$   
 $\frac{d}{dx} (\ln y = \sin x \ln x)$   $\frac{dy}{dx} = x^x (\ln x + 1)$   
 $\frac{1}{y} \frac{dy}{dx} = \cos x \ln x + \frac{\sin x}{x}$

$$\frac{dy}{dx} = y \left( \cos x \ln x + \frac{\sin x}{x} \right) = x^{\sin x} \left( \cos x \ln x + \frac{\sin x}{x} \right)$$

197: 23-26, 32-36

Tuesday, October 2, 2018 9:51 AM

$$\frac{d}{dx} \ln a = \frac{1}{a} \cdot \frac{da}{dx}$$

Tuesday, October 2, 2018 10:00 AM

$$\log_a b = \frac{\log_c b}{\log_c a}$$

$$\log_{10} x = \frac{\ln x}{\ln 10}$$