3.2 examples

Wednesday, September 26, 2018 12:44 PM

3.2 examples

Calculus AB, section 3.2 (The Derivative as a Function)

Example 1

Prove that f(x) = $x^3 - 12x$ is differentiable. Compute $f'(x)$ and find an equation of the tangent line at x = -3.

Example 2 $y = \frac{1}{x^2}$ $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$

Prove that $y = x^{-2}$ is differentiable and calculate y'.

$$= \frac{\frac{x^2}{x^2}\cdot\frac{1}{(x+h)^2} - \frac{1}{x^2}\cdot\frac{(x+h)^2}{(x+h)^2}}{h} = \frac{x^2 - (x+h)^2}{x^2(x+h)^2 h}$$

$$= \frac{x^2 - (x^2 + 2xh + h^2)}{x^2(x+h)^2 h} = \frac{-2x\cancel{h} - h^{\cancel{2}}}{x^2(x+h)^2 \cancel{h}}$$

$$= \frac{-2x - h}{x^2(x+h)^2} \longrightarrow \frac{-2x}{x^4} = \frac{-2}{x^3} \checkmark$$

Wednesday, September 26, 2018 12:45 PM

__Prove Theorem 1__: $\frac{d}{dx}x^n = nx^{n-1}$ (The Power Rule)

13 9; 1,5

$$\frac{d}{dx}x^4 = 4x^3 \qquad \frac{d}{dx}x^{\pi} = \pi x^{\pi - 1}$$

$$\frac{d}{dx}\left(\frac{2}{5}x^{3/5}\right) = \frac{6}{25}x^{-2/5}$$

Prove Theorem 2: Assume that f and g are differentiable functions.
Sum Rule: The function f + g is differentiable and
$(f+g)' = f' + g'$

Constant Multiple Rule: For any constant c, cf is differentiable and
$(cf)' = cf'$

Example 3

Find the points on the graph of $f(t) = t^3 - 12t + 4$, where the tangent line is horizontal.

$f'(t) = 3t^2 - 12 = 0 \quad t = \pm 2$

$f(-2) = -8 + 24 + 4 = 20 \quad (-2, 20)$

$f(2) = 8 - 24 + 4 = -12 \quad (2, -12)$

Example 4

Calculate $\frac{dg}{dt}|_{t=1}$, where g(t) = $t^{-3} + 2\sqrt{t} - t^{-4/5}$

$$\frac{dg}{dt} = -3t^{-4} + 2\left(\frac{1}{2}\right)t^{-1/2} + \frac{4}{5}t^{-9/5}$$

$$= -3 + 1 + \frac{4}{5} = -1\,1/5 = -\frac{6}{5}$$

Example 5 (Graphical Insight)

How is the graph of $f(x) = x^3 - 12x$ related to the derivative $f'(x) = 3x^2 - 12$?

Property of $f'(x)$	Property of the Graph of f(x)
$f'(x) < 0$ for $-2 < x < 2$	f declines
$f'(-2) = f'(2) = 0$	maximum or minimum
$f'(x) > 0$ for $x < -2$ and $x > 2$	f increases

Example 6 (Identifying the Derivative)

The graph of f(x) is shown in Figure 5.
Which of (A) or (B) is the graph of $f'(x)$?

$$\frac{d}{dx} e^x = e^x$$

139: 11, 13, 20, 22, 29, 36, 42

Example 7 (Investigating m_b Numerically)

Investigate m_b numerically for b = 2, 2.5, 3 and 10.

h	$\frac{2^h - 1}{h}$	$\frac{(2.5)^h - 1}{h}$	$\frac{3^h - 1}{h}$	$\frac{10^h - 1}{h}$
0.01				
0.001				
0.0001				
0.00001				

$m_b = 1$ for some number between 2 and 3
This number is e

Example 8

Find the equation of the tangent line to the graph of f(x) = $3e^x - 5x^2$ at x = 2

Example 9 Continuous but not Differentiable

Show that $f(x) = |x|$ is continuous but not differentiable at $x = 0$.

Example 10 Vertical Tangents

Show that f(x) = $x^{1/3}$ is not differentiable at x = 0.

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

$$f'(0) = \frac{1}{0} \quad \text{DNE}$$

Show that $f(x) = x^3$ is differentiable.

Find $f'(x)$ and evaluate $f'(-3)$

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1
\end{array}
$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$= \frac{(x+h)^3 - x^3}{h} = \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$\lim_{h \to 0} = 3x^2 + 3xh + h^2 \longrightarrow 3x^2 = f'(x)$$

$$f'(-3) = 27$$