

3.3 examples

Monday, October 1, 2018 12:52 PM



3.3 examples

AB Calculus, sect. 3.3

Example 1

Find the derivative of $h(x) = 3x^2(5x+1)$

product rule: $h'(x) = 6x(5x+1) + 3x^2(5)$
 $= 30x^2 + 6x + 15x^2 = 45x^2 + 6x$

distribute first: $15x^3 + 3x^2$
 $h'(x) = 45x^2 + 6x$

Example 2

Find the derivative of $y = (x^{-1} + 1)(x^{\frac{3}{2}} + 1)$

$$\begin{aligned}\frac{dy}{dx} &= (-x^{-2})(x^{\frac{3}{2}} + 1) + (x^{-1} + 1)\left(\frac{3}{2}x^{\frac{1}{2}}\right) \\ &= \underline{-x^{-1/2}} - x^{-2} + \underline{\frac{3}{2}x^{-1/2}} + \frac{3}{2}x^{1/2} \\ &\quad \frac{1}{2}x^{-1/2} - x^{-2} + \frac{3}{2}x^{1/2}\end{aligned}$$

Example 3

Calculate $\frac{d}{dt} t^2 e^t$

Prove the Product Rule:

Example 4

Compute the derivative of $f(x) = \frac{x}{x+1}$

$$f'(x) = \frac{(x+1) \cdot 1 - x(1)}{(x+1)^2} = \frac{1}{(x+1)^2}$$

Example 5

$$\begin{aligned}\text{Calculate } \frac{d}{dt} \frac{e^t}{e^t + t} &= \frac{(e^t + t)e^t - e^t(e^t + 1)}{(e^t + t)^2} \\ &= \frac{e^{2t} + te^t - e^{2t} - e^t}{(e^t + t)^2} = \frac{te^t - e^t}{(e^t + t)^2}\end{aligned}$$

Example 6

Find an equation of the tangent line to the graph of $f(x) = \frac{3x^2 - 2}{4x^3 + 1}$

at $x = 1$. $f'(x) = \frac{(4x^3 + 1)(6x) - (3x^2 - 2)(12x^2)}{(4x^3 + 1)^2}$

$$f'(1) = \frac{(5)(6) - (1)(12)}{25} = \frac{18}{25}$$

$$f(1) = \frac{1}{5}$$

$$\boxed{y - \frac{1}{5} = \frac{18}{25}(x - 1)}$$

Example 7 - Power Delivered by a Battery

The power that a battery can supply to an apparatus such as a cell phone depends on the *internal resistance* of the battery. For a battery of voltage V and internal resistance r , the total power delivered to an apparatus of

resistance R is:

$$P = \frac{V^2 R}{(R+r)^2}$$

- a) Calculate $\frac{dP}{dR}$, assuming that V and r are constants.

$$= \frac{(R+r)^2 V^2 - V^2 R \cdot 2(R+r)}{(R+r)^4}$$

$$14f! \quad 9-12, 15, 16$$



$$(f g)' = f' g' \quad \text{X}$$
$$(f g)' = f' g + f g'$$

$$\left(\frac{f}{g}\right)' = \frac{f'}{g}$$

~~X~~

$$\left(\frac{f}{g}\right)' = \frac{g f' - f g'}{g^2}$$

p. 191! 47-50, 53, 55, 71

$$\frac{10 - 5}{10} = \frac{1}{2}$$

$$\frac{\cancel{10} - 5}{\cancel{10}}$$