

3.6 examples

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Calculus AB, section 3.6

Example 1

Calculate $f'(x)$ and $f''(x)$, where $f(x) = x \cos x$

$$a = x$$

$$a' = 1$$

$$b = \cos x$$

$$b' = -\sin x$$

$$f'(x) = \cos x + x(-\sin x) \\ = \cos x - x \sin x$$

$$f''(x) = -\sin x - \sin x - x \cos x$$

$$a = x$$

$$a' = 1$$

$$b = \sin x$$

$$b' = \cos x$$

$$-2 \sin x - x \cos x$$

Example 2

Verify the formula $\frac{d}{dx} \tan x = \sec^2 x$ (Figure 2)

$$\frac{d}{dx} \left(\frac{\sin x}{\cos x} \right) = \frac{\cos x (\cos x) - \sin x (-\sin x)}{\cos^2 x}$$

$$\frac{1}{\cos^2 x} = \sec^2 x$$

Example 3

Find an equation of the tangent line to the graph of $y = \tan \theta \sec \theta$ at

$$\theta = \frac{\pi}{4} \quad y'(x) = \tan \theta (\sec \theta \tan \theta) + (\sec^2 \theta)(\sec \theta)$$

$$\sec \theta \tan^2 \theta + \sec^3 \theta$$

$$y'(\frac{\pi}{4}) = \sec(\frac{\pi}{4}) \tan^2(\frac{\pi}{4}) + \sec^3(\frac{\pi}{4})$$

$$(\sqrt{2})(1) + \sqrt{8}$$

$$\sqrt{2} + \sqrt{8}$$

$$\sqrt{2} + 2\sqrt{2}$$

$$y'(\frac{\pi}{4}) = 3\sqrt{2}$$

$$y - \sqrt{2} = 3\sqrt{2} \left(\theta - \frac{\pi}{4} \right)$$

$$\begin{array}{r} 170 \\ \hline 1, 2, 5, 6, 10, \\ 12, 27, 33 \end{array}$$

Example 4

Calculate $f'(0)$, where $f(x) = e^x \cos x$

Show That $\frac{d}{dx} \sin x = \cos x$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = f'(x) \quad f(x) = \sin x$$

$$\frac{d}{dx} \sin x = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h} = \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h}$$

$$= \lim_{h \rightarrow 0} \sin x \left(\frac{\cos h - 1}{h} \right) + \frac{\cos x \sin h}{h}$$

$$\lim_{h \rightarrow 0} \frac{1 - \cos h}{h} = 0$$

$$= \cos x$$