3.7 examples

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3.7 examples

Calculus AB, section 3.7 (Chain Rule)

Example 1

Calculate the derivative of y = $\sqrt{x^3 + 1}$

Example 2

Calculate $\frac{dy}{dx}$ for: a. $y = \cos(x^2)$

b. $y = \tan\frac{x}{x+1}$

$$\frac{dy}{dx} = \sec^2\left(\frac{x}{x+1}\right)\cdot\left(\frac{x+1-x}{(x+1)^2}\right)$$

$$= \sec^2\left(\frac{x}{x+1}\right)\cdot\frac{1}{(x+1)^2}$$

Example 3

$V = \frac{4}{3}\pi r^3$ 170: 29, 30, 35, 36, 39, 40, 41

Imagine a sphere whose radius r increases at a rate of 3 cm/s. At what rate is the volume V of the sphere increasing when r = 10 cm?

$$\frac{dV}{dt} = \frac{dV}{dr}\cdot\frac{dr}{dt}$$

$t \to r \to V$

$V(r(t))$

$$= 4\pi r^2 \cdot 3$$

$$= 4\cdot\pi\cdot 100\cdot 3 = 1200\pi\ \frac{cm^3}{s}$$

Example 4

Find the derivatives: a. $y = (x^3 + 9x + 2)^{-1/3}$

$$\frac{dy}{dx} = -\frac{1}{3}\left(x^3 + 9x + 2\right)^{-4/3}\left(3x^2 + 9\right)$$

b. $y = \sec^4 t$ $= (\sec t)^4$

$$\frac{dy}{dt} = 4\sec^3 t \cdot \sec t \tan t$$

$$4\sec^4 t \tan t$$

Example 5

Differentiate: a. $f(x) = e^{9x}$

x^n

$$f'(x) = 9e^{9x}$$

b. $f(x) = e^{\cos x}$ $f'(x) = -\sin x \, e^{\cos x}$

Example 6 – Trigonometric Derivatives in Degrees

Calculate the derivative of the sine function as a function of degrees rather than radians. $\underline{\sin}\ x$ = sine with x in degrees

$$\underline{\sin}\ x = \sin\left(\frac{\pi}{180}\cdot x\right)$$

$$\frac{d}{dx}\sin\left(\frac{\pi}{180}\cdot x\right) = \cos\left(\frac{\pi}{180}x\right)\cdot\frac{\pi}{180}$$

$$\underline{\cos}(x)\cdot\frac{\pi}{180}$$

$$\frac{d}{dx}\underline{\sin}\ x = \underline{\cos}\ x\cdot\frac{\pi}{180}$$

Example 7

Find the derivative of $y=\sqrt{x+\sqrt{x^2+1}}$ $=\left(x+(x^2+1)^{1/2}\right)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}\left(x+(x^2+1)^{1/2}\right)^{-1/2}\cdot\left(1+\frac{1}{2}(x^2+1)^{-1/2}\cdot 2x\right)$$

f(x) is the curve below. g(x) has straight line segments.

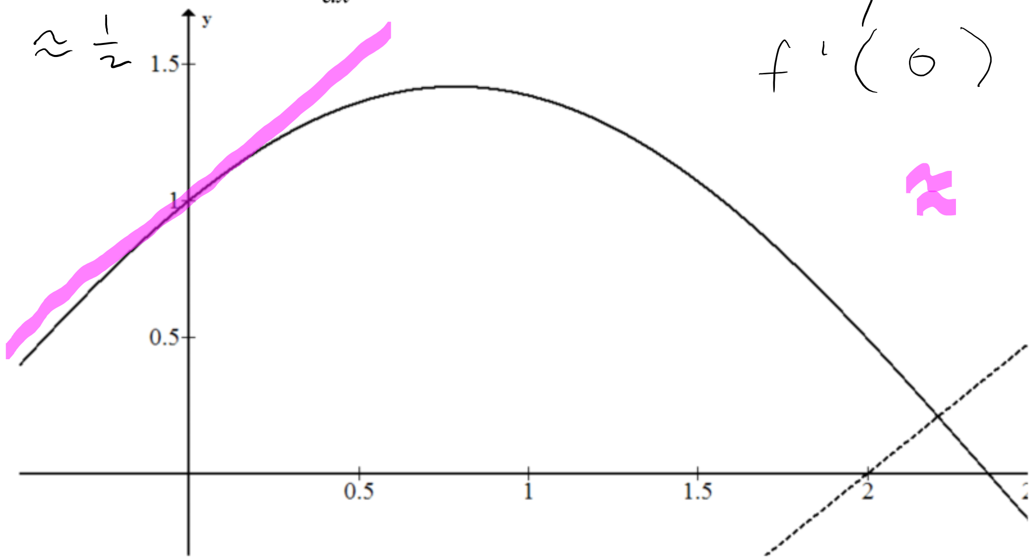
Find $g'(2)$. Estimate $\frac{d}{dx}f(g(x))$ at x = 2.

$= f'(g(x)) \cdot g'(x)$

$f'(0) \cdot g'(2)$

$\approx \frac{1}{2}$

$\approx 1 \cdot \frac{1}{2} = \frac{1}{2}$



x	1	3	5
f(x)	4	0	3
$f'(x)$	2	-1	1
g(x)	3	2	5
$g'(x)$	1	4	2

178: 35, 36, 40, 43, 44, 53, 54, 55

Find:

$\frac{d}{dx}\sin(g(x))$ at x = 3

$\cos(g(x)) \cdot g'(x) = \cos(g(3)) \cdot g'(3)$

$= (\cos 2) \cdot 4$

$\frac{d}{dx}$ f(g(x)) at x = 5

$\frac{d}{dx}$ f(x^2) at x = 1

$$\frac{d}{dx}\left(\text{outside}(\text{inside})\right) = \text{outside}'(\text{inside}) \cdot \text{inside}'$$

$$\frac{d}{dx}\cos\left(x^3\right) = -\sin\left(x^3\right) \cdot 3x^2$$

$$\frac{d}{dx}\sqrt{x^4+1} = \frac{d}{dx}\left(x^4+1\right)^{1/2} = \frac{1}{2}\left(x^4+1\right)^{-1/2} \cdot 4x^3$$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$