3.9 examples

Wednesday, September 26, 2018 10:03 AM

3.9 examples

Calculus AB, section 3.9 (Derivatives of Inverse Functions)

Example 1 $g(5) = 1$ $f'(x) = 2x$ $f'(1) = 2$

Calculate $g'(x)$ where g(x) is the inverse of f(x) = $x^2 + 4$ on the domain $\{x : x \geq 0\}$

$y = x^2 + 4$ $f(1) = 5$

$x = y^2 + 4$

$x - 4 = y^2$

$y = \pm\sqrt{x-4} = g(x)$

$g(x) = (x-4)^{1/2}$

$g'(x) = \frac{1}{2}(x-4)^{-1/2}$

$= \dfrac{1}{2\sqrt{x-4}}$

$g'(5) = \frac{1}{2}$

Example 2 – Calculating $g'(x)$ without solving for g(x)

$f'(x) = 1 + e^x$

Calculate $g'(1)$ where g(x) is the inverse of f(x) = x + e^x

$x + e^x = 1$

$f(0) = 1$

$g(1) = 0$

$$g'(1) = \frac{1}{f'(0)} = \frac{1}{2}$$

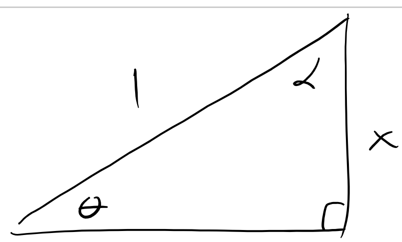
$y = x + e^x$

$x = y + e^y$

$f'(0) = 2$

Example 3 – Complementary Angles

Prove: $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$



$\theta + \alpha = \pi/2$

Example 4

Calculate $f'(\frac{1}{2})$ where f(x) = arcsin(x^2)

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$f'(x) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x$$

$$= \frac{2x}{\sqrt{1-x^4}}$$

$$f'\left(\frac{1}{2}\right) = \frac{1}{\sqrt{1-(\frac{1}{2})^4}}$$

$$= \frac{1}{\sqrt{\frac{15}{16}}} = \frac{4}{\sqrt{15}}$$

191 : 3, 4, 11, 12, 13, 15

$$\frac{d}{dx}\tan^{-1}u(x) = \frac{1}{1+u^2}\cdot\frac{du}{dx}$$

Example 5

Calculate:

a. $\dfrac{d}{dx}\tan^{-1}(3x+1)$ $= \dfrac{3}{1+(3x+1)^2}$

b. $\dfrac{d}{dx}\sec^{-1}(e^x+1)$

$$\frac{d}{dx}\sec^{-1}u = \frac{1}{|u|\sqrt{u^2-1}}\cdot\frac{du}{dx}$$

$$= \frac{1}{|e^x+1|\sqrt{(e^x+1)^2-1}}\cdot e^x$$

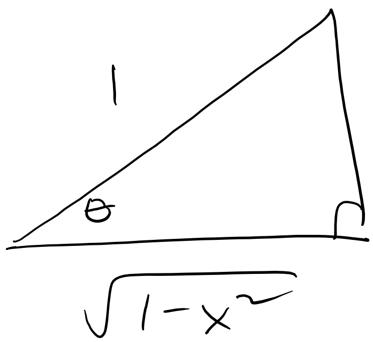
3.9

$g(x) = f^{-1}(x)$

How are their derivatives related?

Thursday, September 27, 2018 9:55 AM

Show That $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$



$$f(x) = \sin x$$

$$g(x) = f^{-1}(x) = \sin^{-1} x$$

$$\frac{d}{dx} g(x) = \frac{d}{dx}\sin^{-1}x = \frac{1}{\frac{d}{dx}\sin(g(x))}$$

$$= \frac{1}{\cos(g(x))} = \frac{1}{\cos(\sin^{-1}x)} = \frac{1}{\cos\theta}$$

$$= \frac{1}{\sqrt{1-x^2}}$$ ✓

191: 18 – 21, 23, 26, 33