4.1 examples

Thursday, October 11, 2018 9:56 AM

4.1 examples

Calculus AB, section 4.1 – Linear Approximation and Applications

$$\Delta f \approx f'(a)\Delta x$$

Example 1

Use the Linear Approximation to estimate $\frac{1}{10.2} - \frac{1}{10}$.

Then find out how accurate your estimate is.

(-0.002)

Example 2 – Position and Velocity

The position of an object in linear motion at time t (in seconds) is $s(t) = t^3 - 20t + 8$ m. Estimate the distance traveled over the time interval [3, 3.025].

($\Delta s = 0.175$ m)

Example 3 – Thermal Expansion

A thin metal cable has length L = 6 in. when the ambient temperature is T = 70˚ F. Estimate the change in length when T rises to 75˚, assuming that

$$\frac{dL}{dT} = kL$$

where $k = 9.6 \times 10^{-6}$ ˚F^{-1} (k is called the coefficient of thermal expansion).

($\Delta L = 2.9 \times 10^{-4}$ in.)

Example 4 – Weight Loss in an Airplane

Newton's Law of Gravitation can be used to show that if an object weighs w pounds on the surface of the earth, then its weight at distance x from the center of the earth is:

$$W(x) = \frac{wR^2}{x^2} \qquad (x \geq R)$$

where R = 3960 miles is the radius of the earth. Estimate the weight lost by a 200-lb football player flying in a jet at an altitude of 7 miles.

($\Delta W = -0.7$ lb.)

Example 5 – Effect of an Inexact Measurement
The Bonzo Pizza Co. claims that its pizzas are circular with diameter 18 in.

a) What is the area of the pizza?

b) Estimate the amount of pizza lost or gained if Bonzo's chefs err in the diameter by at most 0.4 in.

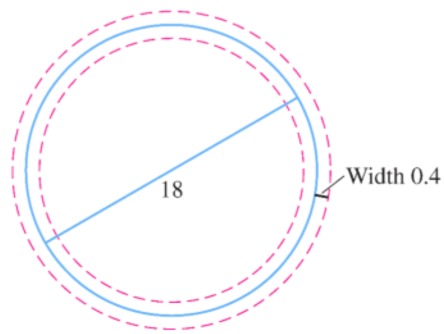

FIGURE 3 The border of the actual pizza lies between the dashed circles.

($\Delta A = \pm\ 3.6\pi = 11.3$ in^2.)

Linearization: $f(x) \approx L(x) = f'(a)(x-a) + f(a)$

Example 6

Compute the linearization of f(x) = $\sqrt{x}e^{x-1}$ at a = 1.

$f(1) = 1$

$f'(x) = \frac{1}{2}x^{-1/2}e^{x-1} + \sqrt{x}e^{x-1}$

$f'(1) = \frac{1}{2} + 1 = 1.5$

$y - 1 = 1.5(x-1)$

$L(x) = 1.5(x-1) + 1$

Use $L(x)$ to estimate $f(1.1)$

$L(1.1) = 1.5(1.1-1) + 1$

$= 1.15$

(L(x) = $\frac{1}{2}x + \frac{1}{2}$)

Example 7

Use the linearization to estimate $\tan(\frac{\pi}{4} + 0.02)$ and compute the percentage error.

$\tan \frac{\pi}{4} = 1$

$\frac{d}{dx} \tan x = \sec^2 x \qquad \sec^2 \frac{\pi}{4} = 2$

$y - 1 = 2(x - \pi/4)$

$L(x) = 2(x - \pi/4) + 1$

$L\left(\frac{\pi}{4} + 0.02\right) = 2\left(\frac{\pi}{4} + 0.02 - \frac{\pi}{4}\right) + 1$

$= 1.04$

$\% \text{ error} = \frac{1.0408 - 1.04}{1.0408}(100)$

(L = 1.04. % error = 0.08%)

Approximate $\sqrt{26}$ and find the % error

$g(x) = \sqrt{x}$ $\quad g'(x) = \frac{1}{2} x^{-1/2}$

$g'(25) = \frac{1}{2} \cdot \frac{1}{\sqrt{25}} = \frac{1}{10}$ $\quad g(25) = 5$

$y - 5 = \frac{1}{10}(x - 25)$

$L(x) = \frac{1}{10}(x - 25) + 5$

$L(26) = 5.1$

$\sqrt{26} = 5.0990$ $\quad$ error =

0.02% error

Approximate $e^{-0.1}$

$h(x) = e^x \quad h'(x) = e^x \quad h'(0) = 1$

$h(0) = 1 \quad y - 1 = 1(x - 0) \quad y - 1 = x$

$y = x + 1 \quad y(-0.1) = -0.1 + 1 = 0.9$

% error = 0.5%

p. 218: 41–49 odds

53, 54, 56, 59