4.2 examples

Wednesday, November 28, 2018 12:23 PM

4.2 examples

Calculus AB, section 4.2

Example 1

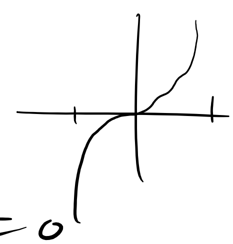
Find the critical points of f(x) = $x^3 - 9x^2 + 24x - 10$.

$$f'(x) = 3x^2 - 18x + 24 = 0$$

$$x^2 - 6x + 8 = 0$$

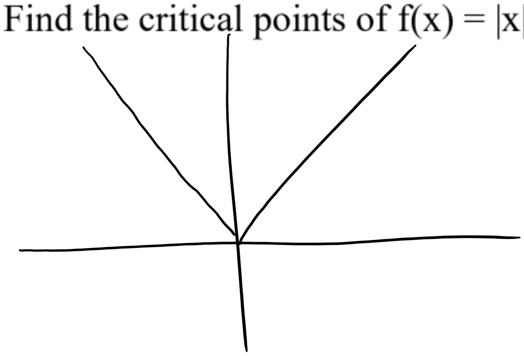
$$(x-4)(x-2) = 0$$

$$x = 2, 4$$



Example 2 – A Nondifferentiable Function

Find the critical points of f(x) = |x|



$x = 0$

$f'(0)$ undefined

or DNE

Example 3

Find the extreme values of $f(x) = 2x^3 - 15x^2 + 24x + 7$ on [0, 6].

$f(0) = 7$ $\quad$ $f(6) = 43$ MAX

$f'(x) = 6x^2 - 30x + 24 = 0$ $\quad$ $f(1) = 18$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$ $\quad$ $f(4) = -23$ MIN

$x = 1, 4$

227: 1, 5, 7, 9, 11 – 14

Example 4 – Function with a Cusp

Find the maximum of f(x) = $1 - (x-1)^{2/3}$ on [-1, 2].

Example 5

Find the extreme values of $f(x) = x^2 - 8\ln x$ on [1, 4].

Example 6 – Trigonometric example

Find the min and max of $f(x) = \sin x \cos x$ on $[0, \pi]$.

Prove: Theorem 4 – Rolle's Theorem

Case 1: If an extreme value occurs at a point c in the open interval (a, b):

Case 2: Both the max and min occur at the endpoints:

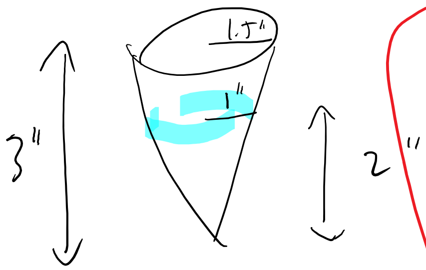
Example 7 – Illustrating Rolle's Theorem

Verify Rolle's Theorem for $f(x) = x^4 - x^2$ on [-2, 2].

Example 8 – Using Rolle's Theorem

Show that f(x) = $x^3 + 9x - 4$ has at most one real root.

Volume of cone $= \frac{1}{3}\pi r^2 h$



DIXIE cup

Water enters at $0.5 \text{ in}^3/s$

height = 2"

Speed at which height is changing at this moment?

$$\frac{r}{h} = \frac{1}{2}$$

$$r = \frac{h}{2}$$

$$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{12}$$

$$\frac{d}{dt}\left(V = \frac{\pi h^3}{12}\right) \longrightarrow \frac{dV}{dt} = \frac{3\pi h^2}{12}\frac{dh}{dt}$$

$$\frac{1}{2} = \frac{3\pi(2)^2}{12}\cdot\frac{dh}{dt}$$

$$\frac{1}{2\pi}\frac{in}{s} = \frac{dh}{dt}$$

$$2.\overline{9} \stackrel{?}{=} 3$$

$$\frac{1}{3} = 0.\overline{3} \qquad \frac{1}{9} = 0.\overline{1}$$

$$1 = 0.\overline{9}$$

$$\frac{d}{dx}\left(xy^2 - x^3y = 6\right)$$

$$x \cdot 2y\frac{dy}{dx} + y^2 - \left(3x^2y + x^3\frac{dy}{dx}\right) = 0$$

$$x2yy' - x^3y' = 3x^2y - y^2$$

$$y'(2xy - x^3) = \ldots$$

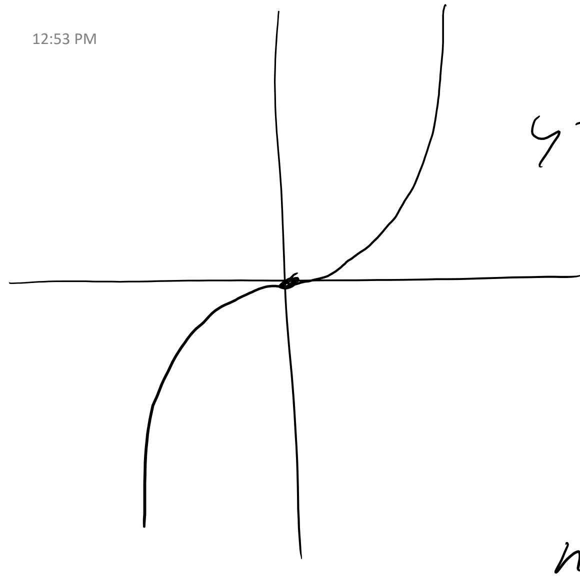
$$y' = \frac{3x^2y - y^2}{2xy - x^3}$$

(c) $2xy - x^3 = 0 \qquad 2xy = x^3$

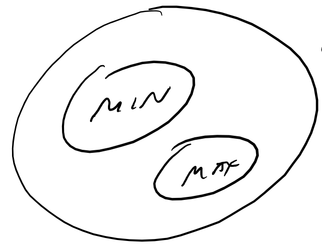
$$y = \frac{x^3}{2x} = \frac{x^2}{2}$$

$$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$$

$$\frac{x^5}{4} - \frac{x^5}{2} = 6$$



$y = x^3$



critical points

critical point at $x = 0$

$y'(0) = 0$

neither a min nor a max