

4.3 examples

Wednesday, December 5, 2018 5:40 AM



4.3 examples

Calculus AB, section 4.3 – The Mean Value Theorem and Monotonicity

Example 1

Illustrate the MVT with $f(x) = \sqrt{x}$ and the points $a = 1$ and $b = 9$.

$$f(1) = 1 \qquad f(9) = 3$$
$$\text{slope of secant line} = \frac{3 - 1}{9 - 1} = \frac{1}{4}$$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} = \frac{1}{4} \qquad x = 4$$

Example 2

Describe the graph of $f(x) = \ln x$. Is $f'(x)$ increasing or decreasing?

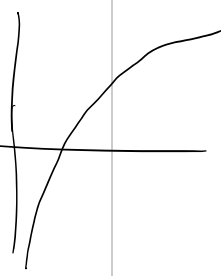
where is $f \downarrow$

where is $f \uparrow$

$f'(x) = \frac{1}{x} > 0 \rightarrow f \uparrow$ everywhere

$f'(x)$ is decreasing

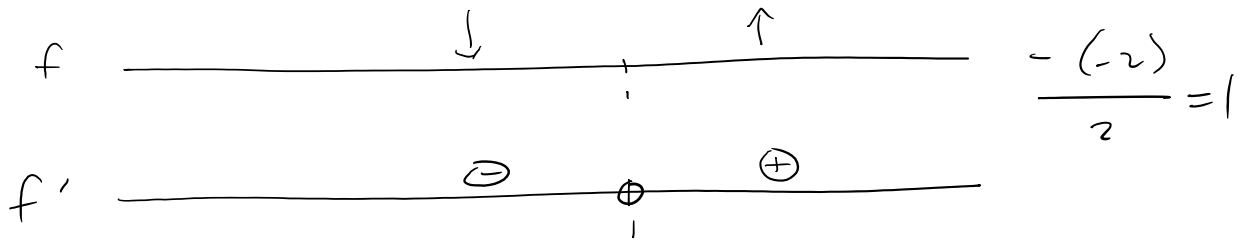
$f''(x) < 0$



Example 3

Find the intervals on which $f(x) = x^2 - 2x - 3$ is monotonic.

$$f'(x) = 2x - 2 = 0 \quad x = 1$$



$f \downarrow$ on $(-\infty, 1)$

$f \uparrow$ on $(1, \infty)$

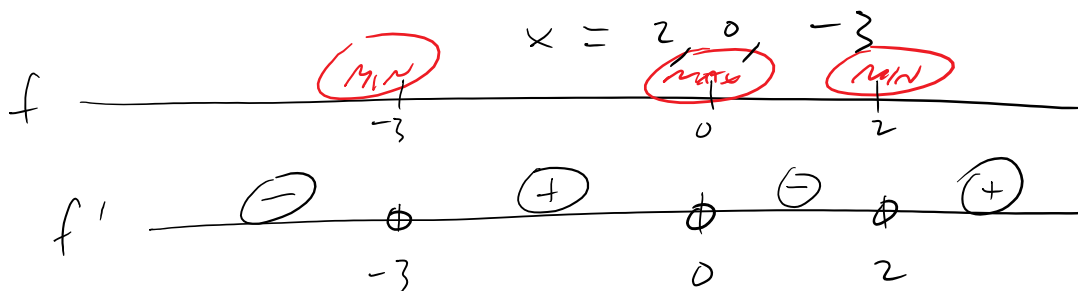
Example 4 – Analyzing critical points

Analyze the critical points of $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - 3x^2 - 7$

$$f'(x) = x^3 + x^2 - 6x = 0$$

$$x(x^2 + x - 6) = 0$$

$$x(x+3)(x-2) = 0$$



Example 5 – Finding Intervals of Increase and Decrease

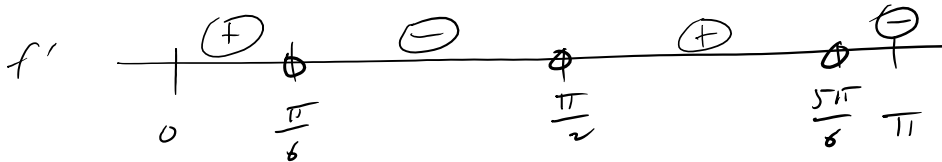
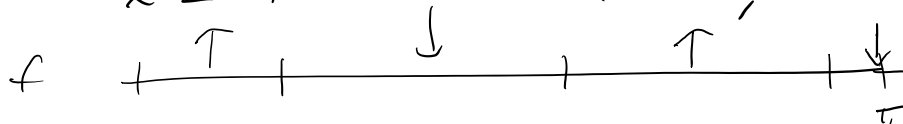
Find the intervals of increase / decrease for $f(x) = \cos^2 x + \sin x$ on $[0, \pi]$.

$$f'(x) = 2 \cos x (-\sin x) + \cos x = 0$$

$$\cos x (-2 \sin x + 1) = 0$$

$$\cos x = 0 \quad -2 \sin x + 1 = 0$$

$$x = \pi/2 \quad x = \pi/6, 5\pi/6$$



f increasing on $(0, \pi/6) + (\pi/2, 5\pi/6)$

f decreasing on $(\pi/6, \pi/2) + (5\pi/6, \pi)$

236: 23, 24, 26, 41, 43, 45

Example 6 – A critical point without a sign transition

Analyze the critical points of $f(x) = \frac{1}{3}x^3 - x^2 + x$.

$$g(x) = \sin x$$

Interval is $[0, \pi/2)$

Find point known by unit to exist

$$\text{Secant line: } \frac{\sin \pi/2 - \sin 0}{\pi/2 - 0} = \frac{1}{\frac{\pi}{2}} = \frac{2}{\pi}$$

$$g'(x) = \cos x = \frac{2}{\pi} \quad x = \cos^{-1}\left(\frac{2}{\pi}\right)$$

$$x \approx 0.881$$

236: 2, 4, 5, 7, 8