

4.3 examples

Tuesday, October 16, 2018 9:42 PM

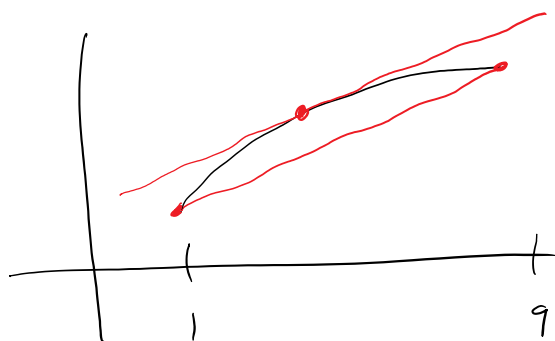


4.3 examples

Calculus AB, section 4.3 – The Mean Value Theorem and Monotonicity

Example 1

Illustrate the MVT with $f(x) = \sqrt{x}$ and the points $a = 1$ and $b = 9$.



$$\frac{\sqrt{9} - \sqrt{1}}{9 - 1} = \frac{2}{8} = \frac{1}{4}$$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{4}$$

$$\frac{1}{2\sqrt{x}} = \frac{1}{4}$$

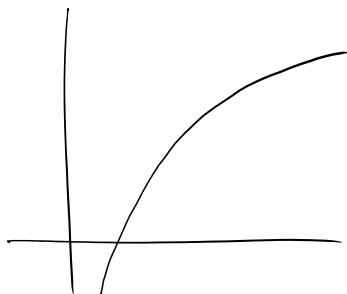
$$x = 4$$

Example 2

Describe the graph of $f(x) = \ln x$. Is $f'(x)$ increasing or decreasing?

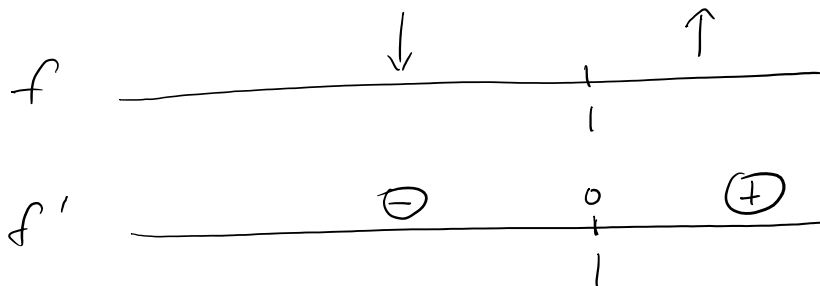
$$f'(x) = \frac{1}{x}$$

$$f'' < 0$$



Example 3

Find the intervals on which $f(x) = x^2 - 2x - 3$ is monotonic.



$$f'(x) = 2x - 2$$

$$f'(1) = 0$$

$$f'(0) = -2$$

$$f'(2) = 2$$

Example 4 – Analyzing critical points

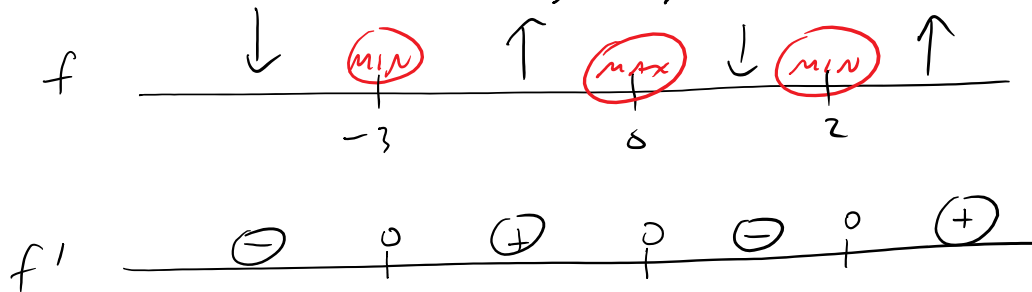
Analyze the critical points of $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - 3x^2 - 7$

$$f'(x) = x^3 + x^2 - 6x = 0$$

$$x(x^2 + x - 6) = 0$$

$$x(x+3)(x-2) = 0$$

$$x = 0, 2, -3$$



p. 236: 2, 5, 7, 8

13, 15-18

Example 5 – Finding Intervals of Increase and Decrease

Find the intervals of increase / decrease for $f(x) = \cos^2 x + \sin x$ on $[0, \pi]$.

$$f'(x) = 2 \cos x (-\sin x) + \cos x = 0$$

$$\cos x (-2 \sin x + 1) = 0$$

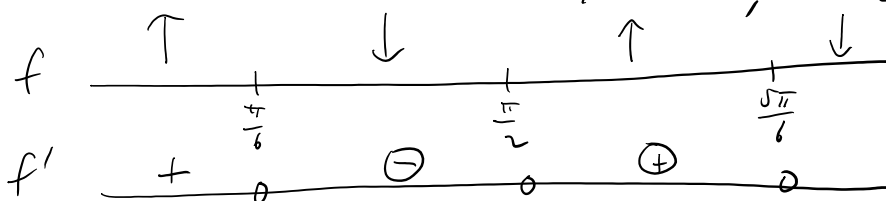
$$\cos x = 0$$

$$x = \pi/2$$

$$-2 \sin x + 1 = 0$$

$$\sin x = 1/2$$

$$x = \pi/6, 5\pi/6$$



f increases on $(0, \pi/6)$ and $(\pi/2, 5\pi/6)$

f decreases on $(\pi/6, \pi/2)$ and $(5\pi/6, \pi)$

Example 6 – A critical point without a sign transition

Analyze the critical points of $f(x) = \frac{1}{3}x^3 - x^2 + x$.

$$f'(x) = x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x = 1$$

$x=1$ not
a min
or max

