4.4 examples

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4.4 examples

Calculus AB, section 4.4 (The Shape of a Graph)

Example 1 – Concavity and Stock Prices

Two stocks, A and B, went up in value and both currently sell for $75 (Figure 3). However, one is clearly a better investment than the other. Explain in terms of concavity. (Assume current trends continue.)

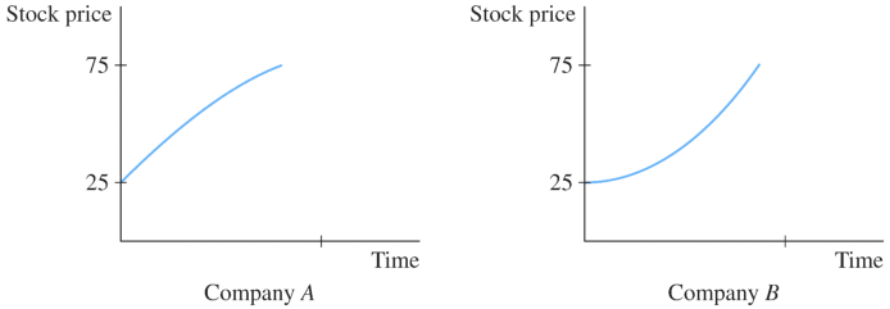


FIGURE 3

Example 2

Find the points of inflection of $f(x) = \cos x$ on $[0, 2\pi]$.

$f'(x) = -\sin x$

$f''(x) = -\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

Example 3 – Finding Points of Inflection and Intervals of Concavity

Find the points of inflection of $f(x) = 3x^5 - 5x^4 + 1$ and determine the intervals where f(x) is concave up and down.

$f'(x) = 15x^4 - 20x^3$

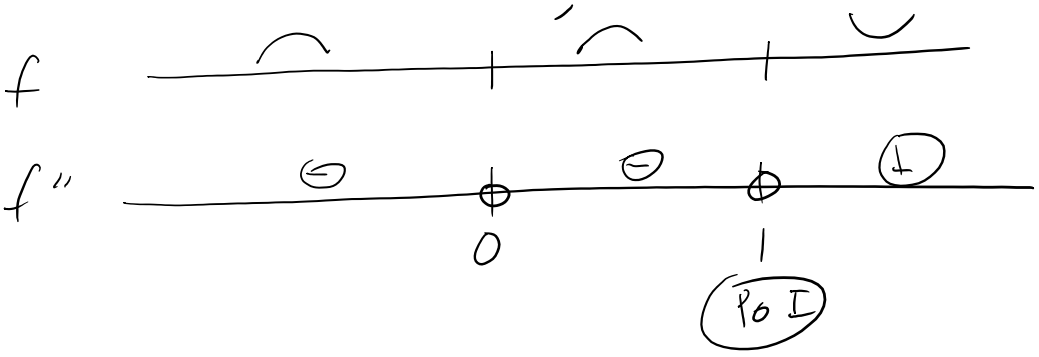
$f''(x) = 60x^3 - 60x^2 = 0$

$60x^2(x-1) = 0$

$x = 0, \quad x = 1$

f is concave up on $(1, \infty)$

f is concave down on $(-\infty, 0) + (0, 1)$



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Example 4 – A case Where the Second Derivative does not exist

Find the points of inflection of $f(x) = x^{5/3}$.

$f'(x) = \frac{5}{3}x^{2/3}$

$f''(x) = \frac{10}{9}x^{-1/3}$ $\quad \frac{10}{9\sqrt[3]{x}} = 0$

$f''(0)$ undefined is $x=0$ POI?

$f''(-1) < 0$ $\quad f''(1) > 1$

So $x=0$ is a POI

$(0, 0)$

Example 5

Analyze the critical points of f(x) = $(2x - x^2)e^x$.

$f'(x) = (2 - 2x)e^x + (2x - x^2)e^x = 0$

$e^x(2 - 2x + 2x - x^2) = 0$

$\rightarrow e^x(2 - x^2) = 0$

$2 - x^2 = 0 \qquad x = \pm\sqrt{2}$

$f''(x) = e^x(-2x) + e^x(2 - x^2)$

$= e^x(-x^2 - 2x + 2)$

$f''(-\sqrt{2}) = e^{-\sqrt{2}}(-(-\sqrt{2})^2 - 2(-\sqrt{2}) + 2)$

$> 0 \quad \therefore f(-\sqrt{2})$ is a MIN

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Example 6 – Second Derivative Test Inconclusive

Analyze the critical points of $f(x) = x^5 - 5x^4$.