4.4 examples

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4.4 examples

Calculus AB, section 4.4 (The Shape of a Graph)

Example 1 – Concavity and Stock Prices
Two stocks, A and B, went up in value and both currently sell for $75 (Figure 3). However, one is clearly a better investment than the other. Explain in terms of concavity.

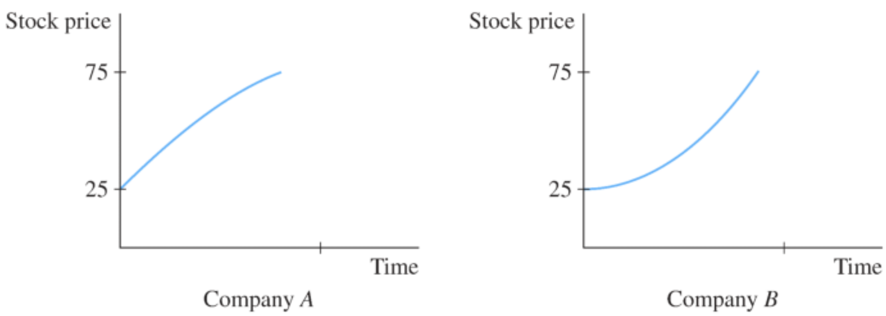


FIGURE 3

Example 2

Find the points of inflection of f(x) = cos x on [0, 2π].

$f'(x) = -\sin x \qquad f''(x) = -\cos x$

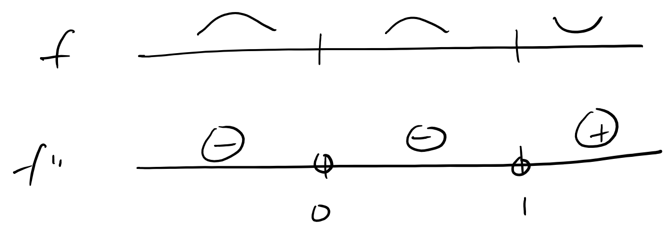
$-\cos x = 0 \qquad x = \pi/2, \; 3\pi/2$

<u>Example 3</u> – Finding Points of Inflection and Intervals of Concavity

Find the points of inflection of f(x) = $3x^5 - 5x^4 + 1$ and determine the intervals where f(x) is concave up and down.

$f'(x) = 15x^4 - 20x^3$ $\qquad f''(x) = 60x^3 - 60x^2$

$60x^3 - 60x^2 = 0$ $\qquad 60x^2(x-1) = 0$ $\qquad x = 0, 1$



PoI at $x=1$ only

Example 4 – A case Where the Second Derivative does not exist

Find the points of inflection of f(x) = $x^{5/3}$.

$f'(x) = \frac{5}{3}x^{2/3}$

$f''(x) = \frac{10}{9}x^{-1/3}$

$= \frac{10}{9\sqrt[3]{x}}$

check out $x = 0$

$f''(-1) < 0$ $f''(1) > 0$

$x = 0$ is POI

Example 5

Analyze the critical points of f(x) = $(2x - x^2)e^x$.

$f'(x) = (2-2x)e^x + (2x-x^2)e^x$

$= e^x(2-x^2) = 0 \quad x = \pm\sqrt{2}$

$f''(x) = (-2x)e^x + (2-x^2)e^x$

$e^x(-x^2-2x+2)$

$f''(-\sqrt{2}) = e^{-\sqrt{2}}\left(-(-\sqrt{2})^2 - 2(-\sqrt{2}) + 2\right)$

$= e^{-\sqrt{2}}(-2 + 2\sqrt{2} + 2) > 0$

$f(-\sqrt{2})$ is a MIN

$f''(\sqrt{2}) = e^{\sqrt{2}}\left(-(\sqrt{2})^2 - 2\sqrt{2} + 2\right) < 0$

$f(\sqrt{2})$ is a MAX

Example 6 – Second Derivative Test Inconclusive

Analyze the critical points of $f(x) = x^5 - 5x^4$.

$f'(x) = 5x^4 - 20x^3 = 0$

$5x^3(x-4) = 0$ $x = 0, 4$ critical points

$f''(x) = 20x^3 - 60x^2$

$f''(0) = 0$ inconclusive

$f''(4) = 20(4)^3 - 60(4)^2 > 0$

$x = 4$ is a MIN

$f'(-1) > 0$ $f'(1) < 0$

$x = 0$ is a MAX

237: 31, 41, 47, 51

243: 15, 20, 22, 31, 33