

4.6 examples

Monday, October 29, 2018 9:57 AM

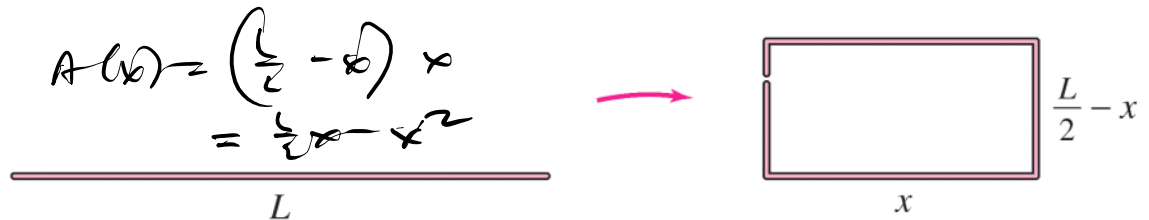


4.6 examples

AB Calculus – section 4.6 – Applied Optimization

Example 1

A piece of wire of length L is bent into the shape of a rectangle. Which dimensions produce the rectangle of maximum area?



$$A(x) = \left(\frac{L}{2} - x\right)x$$
$$= \frac{L}{2}x - x^2$$

FIGURE 2

$$\frac{dA}{dx} = \frac{L}{2} - 2x = 0$$

$$x = \frac{L}{4} \quad \frac{L}{2} - \frac{L}{4} = \frac{L}{4}$$

$$\frac{d^2A}{dx^2} = -2$$

$$243: 41$$

$$265: 2, 3$$

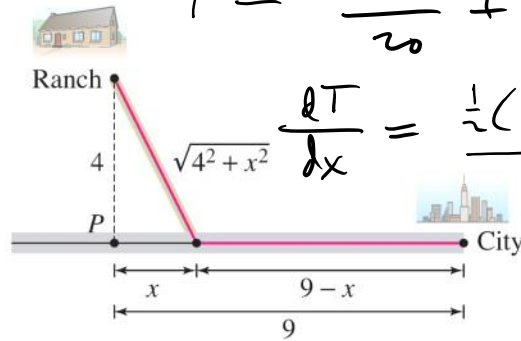
Example 2 – Minimizing Travel Time

Cowboy Clint wants to build a dirt road from his ranch to the highway so that he can drive to the city in the shortest amount of time. The perpendicular distance from the ranch to the highway is 4 miles, and the city is located 9 miles down the highway. Where should Clint join the dirt road to the highway if the speed limit is 20 mph on the dirt road and 55 mph on the highway?

$$D = vT$$

$$T = \frac{D}{v}$$

$$T = \frac{\sqrt{16+x^2}}{20} + \frac{9-x}{55} = \frac{(16+x^2)^{1/2}}{20} + \frac{9-x}{55}$$



$$\frac{dT}{dx} = \frac{\frac{1}{2}(16+x^2)^{-1/2}(2x)}{20} - \frac{1}{55} = 0$$

$$x = 1.561 \text{ miles}$$

FIGURE 3

265: 4, 5, 8
1

Example 3 – Optimization in Agriculture

Experiments show that if fertilizer made from N pounds of nitrogen and P pounds of phosphate is used on an acre of Kansas farmland, then the yield of corn is $B = 8 + 0.3\sqrt{NP}$ bushels per acre. Suppose that the nitrogen costs 25 cents/lb and phosphate costs 20 cents/lb. A farmer intends to spend \$30 per acre on fertilizer. Which combination of nitrogen and phosphate will product the highest yield of corn?

Example 4

Design a cylindrical can of volume 10 ft^3 so that it uses the least amount of metal. In other words, minimize the surface area of the can (including its top and bottom).

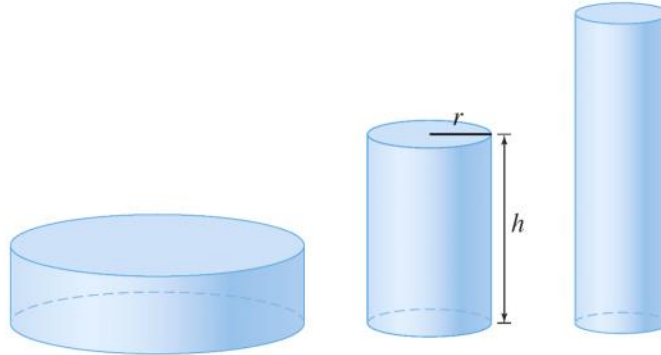


FIGURE 6 Cylinders with the same volume but different surface areas.

Example 5 – Optimization Problem with no Solution

It is possible to design a cylinder of volume 10 ft^3 with the largest possible surface area?