4.7 examples

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4.7 examples

4.7 Examples – L'Hopital's Rule $\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}} = 3$

Example 1

Use L'Hopital's Rule to evaluate $\lim_{x \to 1} \frac{x^3-1}{x-1}$ $\frac{0}{0}$ $\xrightarrow{LR}$ $\frac{3x^2}{1} = 3$

Example 2

Evaluate $\lim_{x \to \pi/2} \frac{\cos^2 x}{\sin x - 1}$ $\quad \frac{\cos^2 \frac{\pi}{2}}{\sin\frac{\pi}{2} - 1} \quad \frac{0}{0}$

$$\xrightarrow{LR} \frac{2\cos x(-\sin x)}{\cos x} = -2\sin x = -2$$

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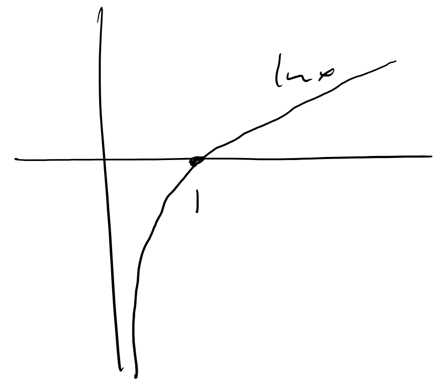
Example 3 – Indeterminate Form ∞/∞

Evaluate $\lim\limits_{x \to 0^+} x\, lnx$ $\quad 0 \cdot -\infty$



rewrite as $\dfrac{\ln x}{\frac{1}{x}}$ $\quad \dfrac{-\infty}{\infty}$

$$= \frac{\ln x}{x^{-1}} \xrightarrow{LR} \frac{\frac{1}{x}}{-x^{-2}} = \frac{\frac{1}{x}}{-\frac{1}{x^2}} = -x \longrightarrow 0$$

Example 4 – Using L'Hopital's Rule Twice

Evaluate $\lim_{x \to 0} \frac{e^x - x - 1}{cosx - 1}$ $\frac{0}{0}$ $\xrightarrow{LR}$ $\frac{e^x - 1}{-\sin x}$ $\frac{0}{0}$

$\xrightarrow{LR}$ $\frac{e^x}{-\cos x} = \frac{1}{-1} = -1$

Example 5 – Assumptions Matter

Can L'Hopital's Rule be applied to $\lim_{x \to 1} \frac{x^2+1}{2x+1}$?

$\frac{2}{3}$

Example 6 – The Form ∞ - ∞

Evaluate $\lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$ $\frac{\sin x}{\sin x}$

$$\frac{x - \sin x}{x \sin x} \quad \frac{0}{0}$$

$$\xrightarrow{LR} \frac{1 - \cos x}{\sin x + x\cos x} \quad \frac{0}{0} \xrightarrow{LR} \frac{\sin x}{\cos x + \cos x - x\sin x}$$

$$= \frac{0}{2} = 0$$

277; 17, 19, 26, 29, 33, 35

Example 7 – The Form 0^0

Evaluate $\lim_{x \to 0^+} x^x$

Comparing Growth of Functions

Example 8

Does $f(x) = x^2$ or $g(x) = x \ln x$ grow faster as $x \to \infty$?

$$\lim_{x \to \infty} \frac{x \ln x}{x^2} = \frac{\ln x}{x} \quad \frac{\infty}{\infty}$$

LR

$$\to \frac{\frac{1}{x}}{1} \to 0 \quad \text{so } x^2 >> x \ln x$$

choose Quick Sort

$(x \ln x)$

Example 9

Jonathan is interested in comparing two computer algorithms whose average run times are approximately $(\ln n)^2$ and $\sqrt{n}$. Which algorithm takes less time for large values of n?

277: 39, 42, 49, 58 – 60, 63