

4.8 examples

Tuesday, January 8, 2019 10:11 AM

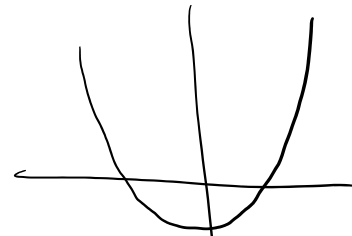


4.8 examples

Calculus AB – section 4.8 (Newton's Method)

$$x^2 - 5 = 0$$

Example 1 – Approximating $\sqrt{5}$



Calculate the first three approximations x_1 , x_2 and x_3 to a root of $f(x) = x^2 - 5$ using the initial guess $x_0 = 2$.

$$f'(x) = 2x$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{-1}{4} = 2\frac{1}{4}$$

$$x_2 = 2.25 - \frac{0.0625}{4.5} = 2.236\bar{1}$$

$$x_3 = 2.236 \dots$$

$$x_4 = 2.236 \dots$$

$$2.236; 5 \rightarrow$$

Example 2

Let c be the smallest positive solution of $\sin 3x = \cos x$.

- a. Use a graphing calculator to choose an initial guess for c .

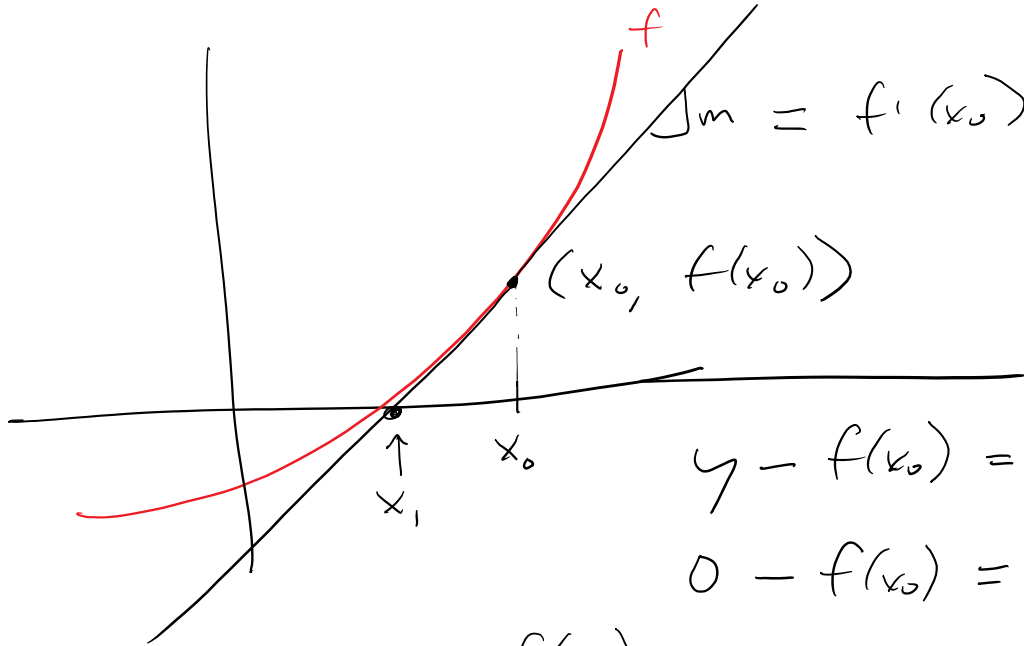
$$y = \sin 3x - \cos x \quad y' = 3 \cos 3x + \sin x$$

$$x_0 = 0.3$$

- b. Use Newton's Method to approximate c to within an error of at most 10^{-6} .

$$x_1 = 0.379$$

Derive The Newton's method formula



$$y - f(x_0) = f'(x_0)(x - x_0)$$

$$0 - f(x_0) = f'(x_0)(x_1 - x_0)$$

$$-\frac{f(x_0)}{f'(x_0)} = x_1 - x_0$$

$$x_0 - \frac{f(x_0)}{f'(x_0)} = x_1$$

Newton's method