4.9 examples

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4.9 examples

Calculus AB – section 4.9 – Antiderivatives

Example 1

Find two antiderivatives of f(x) = cos x.

Then determine the general antiderivative. $\sin x + C$

$F(x) = \sin x$

$F(x) = \sin x + 1$

Example 2

Evaluate $\int 3x^4 - 5x^{2/3} + x^{-3}\, dx$

$\frac{3x^5 - 15x^{5/3}}{5} - \frac{x^{-2}}{2}$

$\frac{3x^5}{5} - 3x^{5/3} - \frac{x^{-2}}{2}$

Example 3

Evaluate $\int \frac{5}{x} - 3x^{-10}\, dx$

$5\ln|x| + \frac{3x^{-9}}{9}$

$5\ln|x| + \frac{1}{3}x^{-9} + C$

Example 4

Evaluate $\int \sin(2t-9) + 20\cos 3t\, dt$

$-\frac{1}{2}\cos(2t-9) + \frac{20}{3}\sin 3t$

Example 5

Evaluate: a) $\int 3e^x - 4\,dx$ $= 3e^x - 4x + C$

$$\int e^{kx}\,dx = \frac{1}{k}e^{kx}$$

b) $\int 12e^{7-3x}\,dx$ $-4e^{7-3x}$

$=$

Example 6

Solve $\frac{dy}{dx} = 4x^7$ subject to the initial condition y(0) = 4

$y = \int 4x^7\,dx = \frac{1}{2}x^8 + C$

$y = \frac{1}{2}x^8 + C$ general solution

$4 = \frac{1}{2}\cdot 0^8 + C$ $C = 4$

$y = \frac{1}{2}x^8 + 4$ particular solution

Example 7

Solve $\frac{dy}{dt} = \sin(\pi t)$, with initial condition y(2) = 2

$y = \int \sin(\pi t)\, dt$

$y = -\frac{1}{\pi}\cos(\pi t) + C$ general

$2 = -\frac{1}{\pi}\cos(2\pi) + C \quad C = 2 + \frac{1}{\pi}$

$y = -\frac{1}{\pi}\cos(\pi t) + 2 + \frac{1}{\pi}$ particular

Example 8 $\frac{dv}{dt} = a \Rightarrow \int a\, dt = v(t)$

At time t = 0, a car traveling with velocity 96 ft/s begins to slow down with constant deceleration a = -12 ft/s^2. Find the velocity v(t) at time t and the distance traveled before the car comes to a halt.

$v(t) = \int a\, dt = \int -12\, dt = -12t + C = v(t)$

$96 = 12(0) + C \quad C = 96 \quad v(t) = -12t + 96$

$\frac{ds}{dt} = v \Rightarrow \int v\, dt = s(t) = \int -12t + 96\, dt$

$s(t) = -6t^2 + 96t + C \leftarrow 0$

$s(0) = 0 \qquad 0 = v(t) = -12t + 96 \quad t = 8$

$s(8) = -6(8)^2 + 96(8) = 384$ ft

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Example 9

Solve $y' = 10e^{-2x}$ with initial condition y(0) = 12

$$\int x^n \, dx = \frac{1}{n+1} \cdot x^{n+1}$$

$$\int x^2 \, dx = \frac{x^3}{3}$$

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Special case

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$$