5-2

$$\int_0^2 e^{-x^3}\,dx \qquad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{-x^3} = 1 - x^3 + \frac{x^6}{2!} - \frac{x^9}{3!} + \frac{x^{12}}{4!} - \frac{x^{15}}{5!} \cdots \frac{x^{3n}}{n!}$$

$$\int e^{-x^3}\,dx = \int 1 - x^3 + \frac{x^6}{2} - \frac{x^9}{6} + \cdots\, dx$$

$$x - \frac{x^4}{4} + \frac{x^7}{14} - \frac{x^{10}}{60} + \frac{x^{13}}{13(4!)} - \frac{x^{16}}{16(5!)} \cdots \frac{x^{3n+1}}{(3n+1)n!}$$

$$= 2 - \frac{2^4}{4} + \frac{2^7}{14} - \frac{2^{10}}{60} + \cdots$$ error < first omitted term

$$\frac{2^{3n+1}}{(3n+1)n!} < 10^{-4}$$ we need 24 terms

2 5th term $< 10^{-4}$

11 $\int_0^1 \tan^{-1}(x^2)\,dx$ $f(x) = \dfrac{1}{1+x^2} = \dfrac{1}{1--x^2}$

$$\int \frac{1}{1+x^2}\,dx = \int 1 - x^2 + x^4 - x^6 \cdots dx \qquad 1 - x^2 + x^4 - x^6 \cdots$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} \cdots$$

$$\tan^{-1} x^2 = x^2 - \frac{x^6}{3} + \frac{x^{10}}{5} - \frac{x^{14}}{7} \cdots \qquad \frac{x^{4n+2}}{2n+1}$$

$$\int \tan^{-1} x^2 = \frac{x^3}{3} - \frac{x^7}{21} + \frac{x^{11}}{55} - \frac{x^{15}}{105} - \cdots \frac{x^{4n+3}}{(2n+1)(4n+3)}\Bigg|_0^1$$

$$= \frac{1^{4n+3}}{(2n+1)(4n+3)} = \frac{1}{(2n+1)(4n+3)} < 10^{-4}$$

$$(2n+1)(4n+3) > 10^4$$

<u>34 terms</u>