7.6

Wednesday, May 29, 2019 9:44 AM

(12) $\int \frac{3x+5}{x^2-4x-5}\,dx = \frac{3x+5}{(x-5)(x+1)}$

$$\frac{3x+5}{(x-5)(x+1)} = \frac{A}{x-5} + \frac{B}{x+1}$$

$$3x+5 = A(x+1) + B(x-5)$$

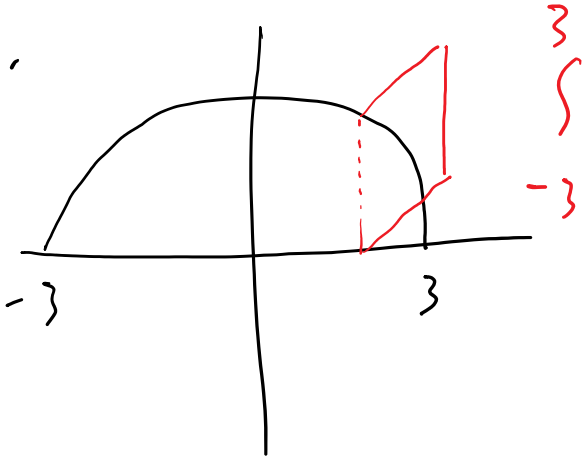
Let $x=-1$: $2 = B(-6)$ $\quad B = -1/3$

Let $x=5$: $20 = 6A$ $\quad A = \frac{20}{6} = \frac{10}{3}$

$$\frac{10}{3}\int \frac{dx}{x-5} - \frac{1}{3}\int \frac{dx}{x+1} = \frac{10}{3}\ln|x-5| - \frac{1}{3}\ln|x+1| + C$$

The base of a solid is the semicircle $y = \sqrt{9-x^2}$ for $-3 \le x \le 3$. The cross-sections $\perp$ to x-axis are $\square$. Find its volume.

Cross-sectional area: $\left(\sqrt{9-x^2}\right)^2 = 9-x^2$

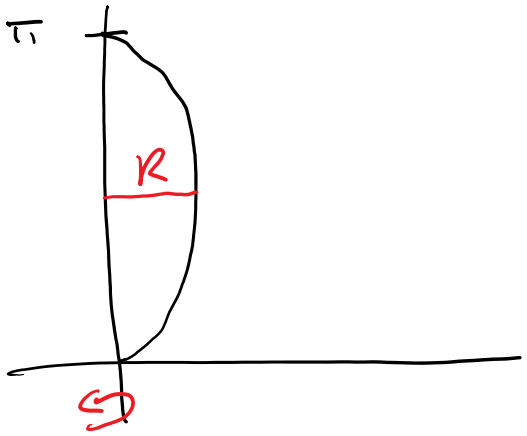


$$\int_{-3}^{3} 9-x^2\,dx$$

$\pi \int R^2$

Rotate $x = \sin y$ about y-axis

$x = 0 \qquad 0 \le y \le \pi$



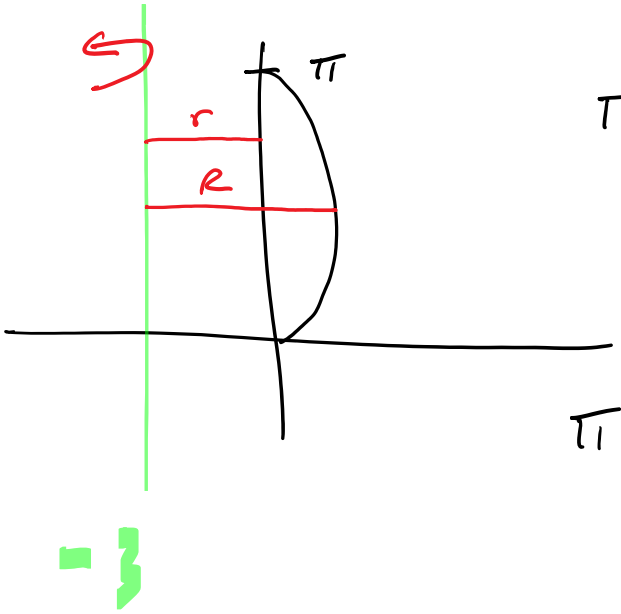
x	y
0	0
0	π
1	$\pi/2$

$$\pi \int_0^{\pi} \sin y \, dy$$

$$= \pi \left(-\cos y \right) \Big|_0^{\pi} = 2\pi$$

Same problem - rotate around

$x = -3$



$\pi \int R^2 - r^2$ (washer)

$R = 7 + \sqrt{\sin y}$

$r = 3$

$\pi \int_0^{\pi} (3 + \sqrt{\sin y})^2 - 3^2 \, dy$

$= 3$