6.2 average of a function

Thursday, May 30, 2019 9:23 AM

$$= \frac{1}{b-a}\int_a^b f(x)\,dx$$

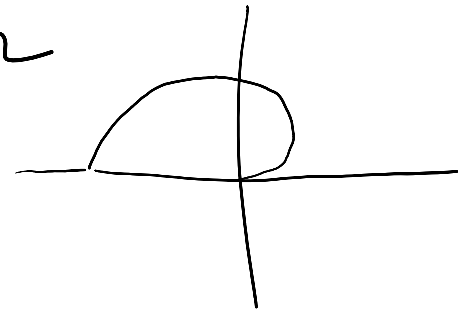
Find

Polar: area + arc length

area: $\frac{1}{2}\int_a^b r(\theta)^2\, d\theta$

arc length: $\int_a^b \sqrt{r(\theta)^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

For $r=\theta$, find the area inside the graph in quadrants 1 + 2



$$\frac{1}{2}\int_0^\pi \theta^2\, d\theta = \frac{1}{2}\cdot\frac{1}{3}\theta^3\Big|_0^\pi$$

$$= \frac{1}{6}\cdot\pi^3$$

Find its arc length

$$\int_0^\pi \sqrt{\theta^2 + 1^2}\, d\theta = 6.101$$

Find the equation of the tangent line to $r=\theta$ at $\theta = \frac{3\pi}{4}$

$r = \frac{3\pi}{4}$ $\quad y = r\sin\theta = \frac{3\pi}{4}\cdot\sin\frac{3\pi}{4} = \frac{3\pi}{4}\cdot\frac{1}{\sqrt{2}}$

$x = r\cos\theta = \frac{3\pi}{4}\cos\frac{3\pi}{4} = -\frac{3\pi}{4}\cdot\frac{1}{\sqrt{2}}$

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $\quad \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$

$$= 1 \cdot \sin 3\pi/4 + 3\pi/4 \cdot -\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} - \frac{3\pi}{4\sqrt{2}}$$

$$\frac{dx}{d\theta} = \frac{dr}{d\theta} \cos\theta - r \sin\theta = 1 \cdot -\frac{1}{\sqrt{2}} - \frac{3\pi}{4} \cdot \frac{1}{\sqrt{2}}$$

$$\frac{dy}{dx} = \frac{\frac{1}{\sqrt{2}} - \frac{3\pi}{4\sqrt{2}}}{-\frac{1}{\sqrt{2}} - \frac{3\pi}{4\sqrt{2}}} = 0.404 \qquad y - \frac{3\pi}{4\sqrt{2}} = 0.404\left(x + \frac{3\pi}{4\sqrt{2}}\right)$$

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Taylor Polynomials

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

Write a Taylor polynomial for

$g(x) = x^2 \sin x \qquad x^2\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots\right)$

$= x^3 - \frac{x^5}{3!} + \frac{x^7}{5!} - \cdots$

$h(x) = \sin 3x \qquad 3x - \frac{(3x)^3}{3!} + \frac{(3x)^5}{5!} - \cdots$