

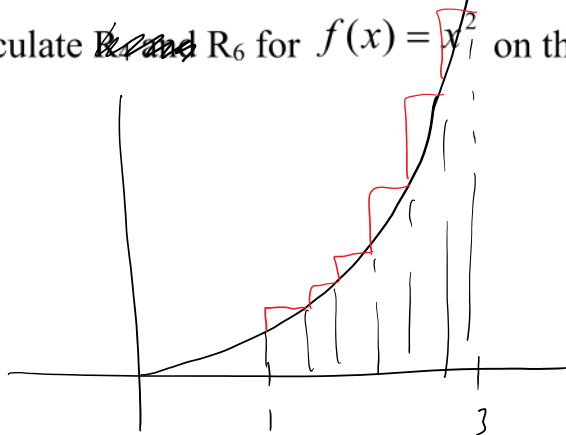


## Calculus AB, section 5.1 (Approximating and Computing Area)

Example 1 – Computing Right-Endpoint Approximations

$$R_6 = R_{RAM_6}$$

Calculate ~~RAM~~  $R_6$  for  $f(x) = x^2$  on the interval  $[1, 3]$ .



$$\begin{aligned} & \left(\frac{4}{3}\right)^2 \cdot \frac{1}{3} + \left(\frac{5}{3}\right)^2 \cdot \frac{1}{3} + \left(\frac{6}{3}\right)^2 \cdot \frac{1}{3} \\ & + \left(\frac{7}{3}\right)^2 \cdot \frac{1}{3} + \left(\frac{8}{3}\right)^2 \cdot \frac{1}{3} + \left(\frac{9}{3}\right)^2 \cdot \frac{1}{3} \\ & = 10.037 \end{aligned}$$

This is an overestimate

### Example 2

Calculate  $R_4$ ,  $L_4$  and  $M_4$  for  $f(x) = x^{-1}$  on  $[2, 4]$ .

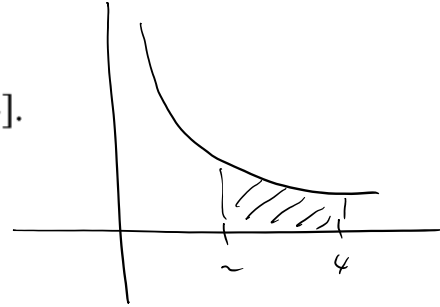
$$R_4 = \frac{1}{2} \left( \frac{1}{2.5} + \frac{1}{3} + \frac{1}{3.5} + \frac{1}{4} \right)$$

$$= 0.634$$

$$M_4 = \frac{1}{2} \left( \frac{1}{2.25} + \frac{1}{2.75} + \frac{1}{3.25} + \frac{1}{3.75} \right) = 0.691$$

$$L_4 = \frac{1}{2} \left( \frac{1}{2} + \frac{1}{2.5} + \frac{1}{3} + \frac{1}{3.5} \right) = 0.759$$

So f: 2, 3, 14, 17



**Example 3 – Calculating Area as a Limit**

Calculate the area under the graph of  $f(x) = x$  over  $[0, 4]$  in three ways:

a.  $\lim_{N \rightarrow \infty} R_N$

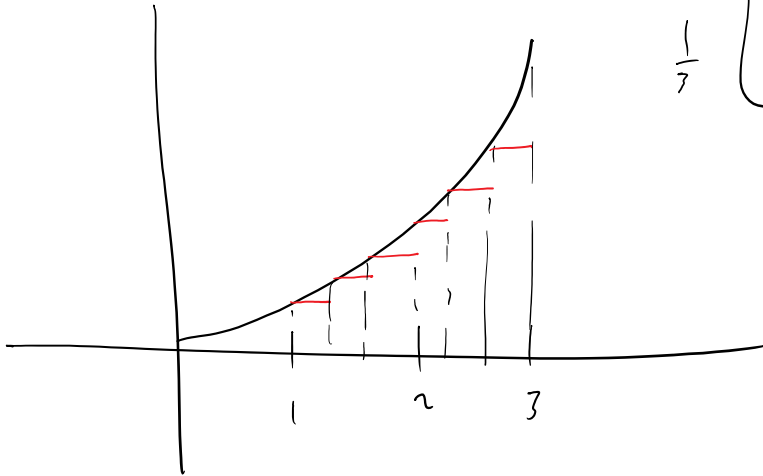
b.  $\lim_{N \rightarrow \infty} L_N$

c. Using geometry

Example 4

Let  $A$  be the area under the graph of  $f(x) = 2x^2 - x + 3$  over  $[2, 4]$ . Find a formula for  $R_N$  and compute  $A$  as the limit  $\lim_{N \rightarrow \infty} R_N$ .

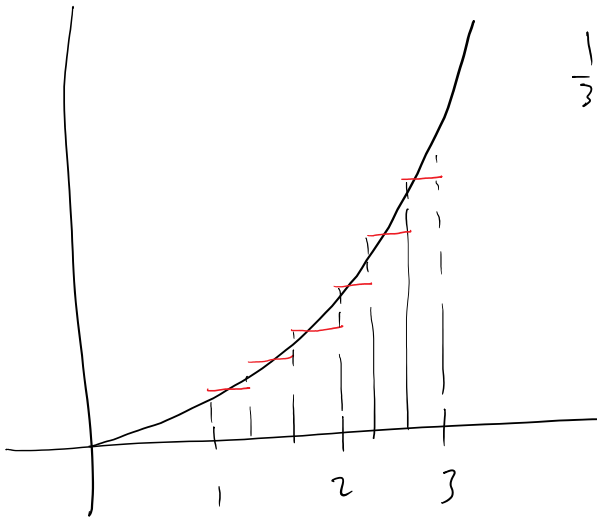
do  $L_b$  on  $y = x^2$  on  $[1, 3]$



$$\frac{1}{7} \left[ 1^2 + \left(\frac{4}{3}\right)^2 + \left(\frac{5}{3}\right)^2 + 2^2 + \left(\frac{7}{3}\right)^2 + \left(\frac{4}{3}\right)^2 \right]$$

$$= 7.370$$

Dis Mc on  $f(x) = x^2$  on  $[1, 3]$



$$\frac{1}{3} \left[ \left(\frac{7}{6}\right)^2 + \left(\frac{9}{6}\right)^2 + \left(\frac{11}{6}\right)^2 + \left(\frac{13}{6}\right)^2 + \left(\frac{15}{6}\right)^2 + \left(\frac{17}{6}\right)^2 \right]$$
$$= 8.648$$

$$\frac{10.037 + 7.37}{2} = 8.7035$$