



Calculus AB: section 5.2 – Definite Integral

Example 1

Let $f(x) = 8 + 12 \sin x - 4x$. Calculate $R(f, P, C)$ for the partition P of $[0, 4]$:

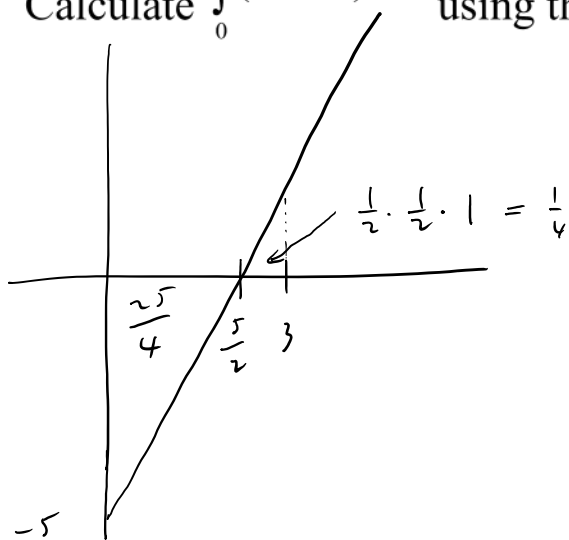
$$P: x_0 = 0 < x_1 = 1 < x_2 = 1.8 < x_3 = 2.9 < x_4 = 4$$

and intermediate points $C = \{0.4, 1.2, 2, 3.5\}$

What is the norm $\|P\|$?

Example 2 – Calculating Signed Area

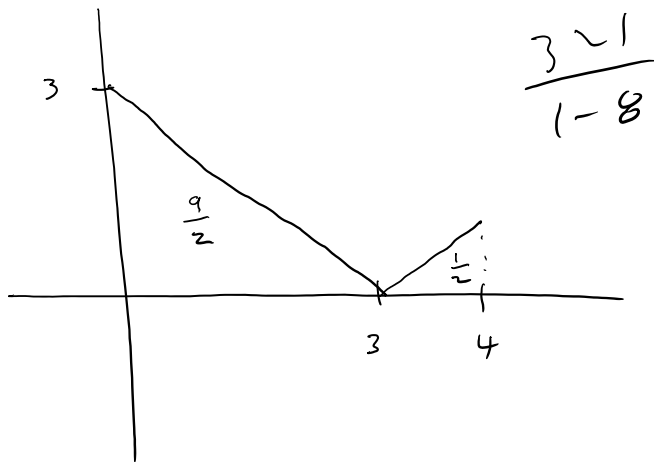
Calculate $\int_0^3 (2x - 5) dx$ using the interpretation as signed area.



$$\int_0^3 (2x - 5) dx = -\frac{25}{4} + \frac{1}{4} = -6$$

Example 3 – Integral of an Absolute Value

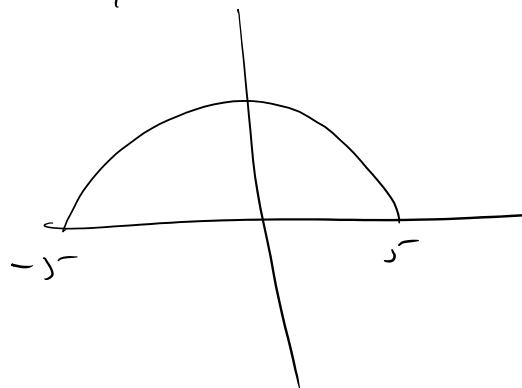
Calculate $\int_0^4 |3-x| dx = 5$



$$\frac{3 \cdot 1}{1-8}$$

$$x^2 + y^2 = 25$$

$$y = \sqrt{25 - x^2}$$



322: 13, 14, 50 - 54, 59-62

Example 4 – The Integral of $f(x) = e^x$

Prove the formula $\int_0^b e^x dx = e^b - 1$

Example 5

Use the limit of right-endpoint approximations to prove that for $b > 0$

$$\int_0^b x^2 dx = \frac{b^3}{3}$$

Example 6

Prove for all b (positive or negative)

$$\int_0^b x \, dx = \frac{1}{2}b^2$$

Example 7

Calculate $\int_4^7 x^2 dx$ using the formula $\int_0^b x^2 dx = \frac{b^3}{3}$ from Example 5.

Example 8

Prove the inequalities:

$$\int_1^5 \frac{1}{x^2} dx \leq \int_1^5 \frac{1}{x} dx$$

$$\frac{3}{4} \leq \int_{1/2}^2 \frac{1}{x} dx \leq 3$$