5.3 examples

Monday, February 4, 2019 12:45 PM

5.3 examples

Calculus AB, section 5.3, The Fundamental Theorem of Calculus, Part 1

Example 1

Calculate the area under the graph:

a) $f(x) = x^3$ over $[2, 4]$ $\int_2^4 x^3\,dx$ $F(x) = \frac{1}{4}x^4$

$$\int_2^4 x^3\,dx = \frac{1}{4}x^4\Big|_2^4 = \frac{1}{4}(4)^4 - \frac{1}{4}(2)^4$$

$$= 64 - 4 = 60$$

b) $g(x) = x^{-3/4} + 3x^{5/3}$ over $[1, 3]$

$$\int x^{-3/4} + 3x^{5/3}\,dx = 4x^{1/4} + \frac{9}{8}x^{8/3}\Big|_1^3$$

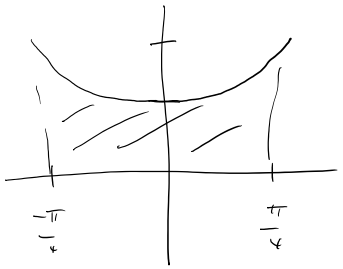
$$= 4(3)^{1/4} + \frac{9}{8}(3)^{8/3} - \left[4(1)^{1/4} + \frac{9}{8}(1)^{8/3}\right]$$

$4(3)^{1/4} + \frac{9}{8}(3)^{8/3} - 5\frac{1}{8}$ 21.2001

Example 2

Sketch the region under $y = \sec^2 x$ for $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ and find its area.

x	y
$-\frac{\pi}{4}$	2
0	1
$\frac{\pi}{4}$	2



$$\int_{-\pi/4}^{\pi/4} \sec^2 x \, dx$$

$$\tan x \Big|_{-\pi/4}^{\pi/4}$$

$$= \tan \pi/4 - \tan(-\pi/4) = 1 - -1 = 2$$

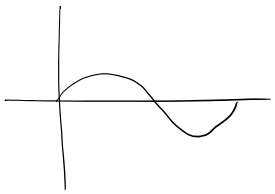
Example 3

Evaluate:

a) $\int_0^{2\pi/3} \sin x \, dx$ $= -\cos x \Big|_0^{2\pi/3} = -\left(\cos \frac{2\pi}{3} - \cos 0\right)$

$= -\left(-\frac{1}{2} - 1\right)$

$= 1.5$

b) $\int_0^{2\pi} \sin x \, dx$  $= 0$

Example 4 – Integral of the Exponential Function

Evaluate $\int_{-2}^{1} e^x \, dx$ $= e^x \Big|_{-2}^{1} = e - \frac{1}{e^2}$

Example 5 – The Logarithm as an Antiderivative

Evaluate:

a) $\int_2^8 \frac{dx}{x}$ $= \ln|x| \Big|_2^8 = \ln 8 - \ln 2 = \ln \frac{8}{2}$

$= \ln 4$

b) $\int_{-4}^{-2} \frac{dx}{x}$ $= \ln|x| \Big|_{-4}^{-2} = \ln 2 - \ln 4$

$= \ln \frac{1}{2}$

329: 10, 20, 29, 40, 43, 45, 51