



## 5.3 examples

## Calculus AB, section 5.3, The Fundamental Theorem of Calculus, Part 1

### Example 1

Calculate the area under the graph:

a)  $f(x) = x^3$  over  $[2, 4]$

$$\int_2^4 x^3 dx = \left. \frac{1}{4} x^4 \right|_2^4$$

$$= \frac{1}{4} \cdot 4^4 - \frac{1}{4} \cdot 2^4 = 64 - 4 = 60$$

$$= \frac{1}{4} \cdot 4^4 + C - \left( \frac{1}{4} \cdot 2^4 + C \right)$$

b)  $g(x) = x^{-3/4} + 3x^{5/3}$  over  $[1, 3]$

$$\int_1^3 x^{-3/4} + 3x^{5/3} dx$$

$$\left. 4x^{1/4} + \frac{9}{8} x^{8/3} \right|_1^3$$

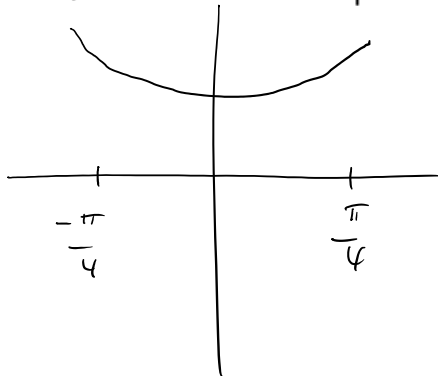
$$4 \cdot 3^{1/4} + \frac{9}{8} \cdot 3^{8/3} - \left( 4 \cdot 1^{1/4} + \frac{9}{8} \cdot 1^{8/3} \right)$$

$$= 4 \cdot 3^{1/4} + \frac{9}{8} \cdot 3^{8/3} - \left( 5 \frac{1}{8} \right) = 21.200$$

### Example 2

Sketch the region under  $y = \sec^2 x$  for  $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$  and find its area.

x	y
$-\frac{\pi}{4}$	2
0	1
$\frac{\pi}{4}$	2



$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x \, dx =$$

$$\tan x \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

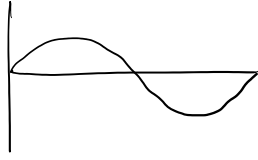
$$= \tan \frac{\pi}{4} - \tan \left( -\frac{\pi}{4} \right)$$

$$= 2$$

### Example 3

Evaluate:

$$\begin{aligned} \text{a) } \int_0^{\pi} \sin x \, dx &= -\cos x \Big|_0^{\pi} = -(\cos \pi - \cos 0) \\ &= -(-1 - 1) \\ &= 2 \end{aligned}$$

$$\text{b) } \int_0^{2\pi} \sin x \, dx = 0$$


Example 4 – Integral of the Exponential Function

Evaluate  $\int_{-2}^1 e^x dx = e^x \Big|_{-2}^1 = e - e^{-2}$

292: 47, 49, 52, 54, 57, 58, 61

### Example 5 – The Logarithm as an Antiderivative

Evaluate:

$$\text{a) } \int_2^8 \frac{dx}{x} = \ln|x| \Big|_2^8 = \ln 8 - \ln 2 = 1.386 \\ = \ln 4$$

$$\text{b) } \int_{-4}^{-2} \frac{dx}{x} = -0.693 = \ln|x| \Big|_{-4}^{-2} \\ = \ln 2 - \ln 4 = \ln 2 - 2\ln 2 \\ = -\ln 2 = \ln \frac{1}{2}$$

329: 9, 19, 27, 29, 31, 35, 37

$$\text{Find } \int (x^{-3/4} + 3x^{5/3}, x, 1, 3)$$

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on your test paper:

$$\int_1^3 (x^{-3/4} + 3x^{5/3}) dx = 21.200$$