5.4 examples

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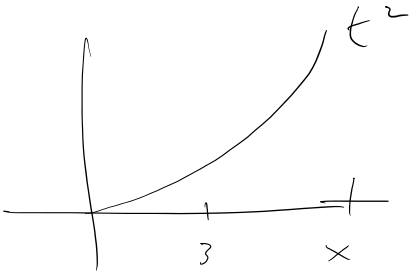
5.4 examples

Calculus AB: section 5.4
Fundamental Theorem of Calculus, part 2



Example 1

Find a formula for the area function $A(x) = \int_3^x t^2\,dt$

$\frac{1}{3}t^3\Big|_3^x = \frac{1}{3}x^3 - 9$

e^{x^2}

Example 2 – Expressing an Antiderivative as an Integral
There is no elementary formula for an antiderivative of $f(x) = \sin(x^2)$. Express the antiderivative $F(x)$ satisfying $F(-\sqrt{\pi}) = 0$ as an integral.

$$F(x) = \int_{-\sqrt{\pi}}^{x} \sin(t^2)\, dt$$

$$\int_a^a f(t)\, dt = 0$$

Example 3 – Differentiating an Integral

$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

Find the derivative of $A(x) = \frac{d}{dx}\int_2^x \sqrt{1+t^3}\,dt$ and calculate $A'(2)$, $A'(3)$ and A(2).

$\frac{dA}{dx} = \sqrt{1+x^3}$

$A'(2) = 3$

$A'(3) = \sqrt{28}$

$A(2) = 0$

Example 4 – Combining the FTC and the Chain Rule

Find the derivative of $G(x) = \int_{-2}^{x^2} \sin t \, dt$

$H(x) = \int_{-2}^{x} \sin t \, dt$

$H'(x) = \sin x$

$H(x^2) = \int_{-2}^{x^2} \sin t \, dt$

$\frac{d}{dx} H(x^2) = (\sin x^2) \cdot 2x$

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$J(x) = \int_{1}^{x} t^2 \, dt$

Find $J'(x) = \frac{d}{dx} \int_{1}^{x} t^2 \, dt = x^2$

Find $J'(\sin x) = \frac{d}{dx} \int_{1}^{\sin x} t^2 \, dt = \sin^2 x \cdot \cos x$

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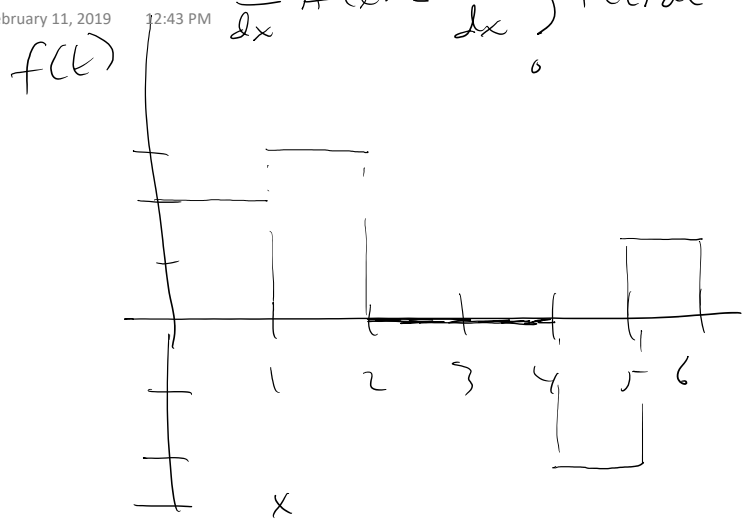
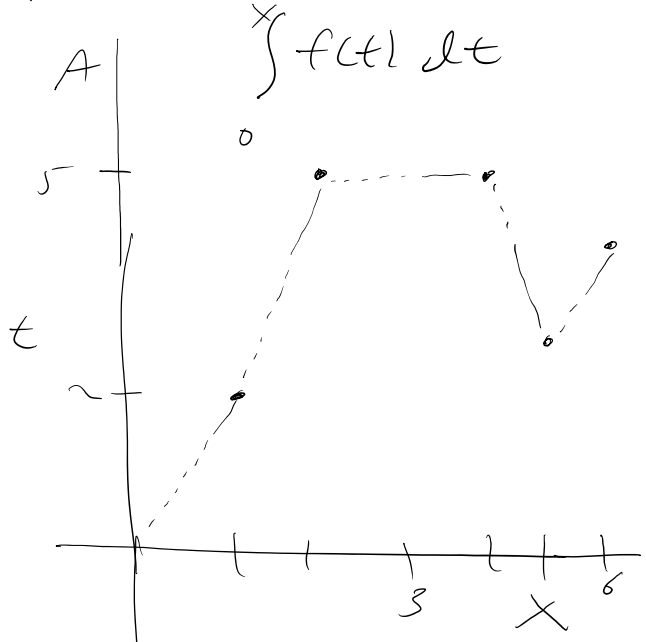
$$\frac{d}{dx} A(x) = \frac{d}{dx} \int_0^x f(t)\,dt = f(x)$$

Sketch $A(x) = \int_0^x f(t)\,dt$





$$A'\left(\frac{1}{2}\right) = \frac{d}{dx} \int_0^{\frac{1}{2}} f(t)\,dt \quad \text{when } x = \frac{1}{2}$$

$$2 = \qquad = f\left(\frac{1}{2}\right)$$

$$A'(3) = \frac{d}{dx} \int_0^x f(t)\,dt \quad \text{when } x = 3 \quad = f(3)$$

$$0 = \qquad f(3)$$

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$$A(x) = \int_0^x \sqrt{\sin t + \pi}\, dt \qquad \frac{dA}{dx} = \sqrt{\sin x + \pi}$$

Find $A'(0) = \sqrt{\pi}$

$A'(\pi) = \sqrt{\pi}$

$A'(2\pi) = \sqrt{\pi}$

$A(0) = 0$