5.6 examples

Monday, February 25, 2019 12:46 PM

5.6 examples

Calculus AB: U-Substitution (5.6)

Example 1

Evaluate $\int 3x^2 \sin(x^3)\, dx$

Choose $u = x^3$

Choose u so that its derivative is also in the integrand.

$u = x^3$

$\frac{du}{dx} = 3x^2$

$du = 3x^2 dx$

substitution

$\int \sin u \, du$

$-\cos u + C$

$-\cos x^3 + C$

Monday, February 25, 2019 12:47 PM

Example 2

Evaluate $\int 2x(x^2+9)^5\,dx$

$u = x^2 + 9$

$du = 2x\,dx$

$\int u^5\,du$

$\frac{(x^2+9)^6}{6} + c$

Monday, February 25, 2019 12:47 PM

Example 3 – Integral of tan θ

Evaluate $\int \tan\theta \, d\theta$

$$\int \frac{\sin\theta}{\cos\theta} \, d\theta$$

$u = \sin\theta$

$du = \cos\theta \, d\theta$

WALL

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$-du = \sin\theta \, d\theta$

$$-\int \frac{du}{u}$$

$$= -\ln|u|$$

$$= -\ln|\cos\theta|$$

$$= \ln|\cos\theta|^{-1}$$

$$= \ln\left|\frac{1}{\cos\theta}\right| = \ln|\sec\theta|$$

Monday, February 25, 2019 12:47 PM

Example 4 – Multiplying du by a Constant

Evaluate $\int \frac{x^2+2x}{(x^3+3x^2+9)^4}\,dx$

$u = x^3+3x^2+9$

$du = 3x^2+6x\,dx$

$\frac{1}{3}du = x^2+2x\,dx$

$\frac{1}{3}\int \frac{1}{u^4}\,du$

$\frac{1}{3}\int u^{-4}\,du$

$\frac{1}{3}\cdot\frac{u^{-3}}{-3}$

$-\frac{1}{9}\left(x^3+3x^2+9\right)^{-3}+C$

Monday, February 25, 2019 12:47 PM

Example 5

Evaluate $\int \sin(7\theta + 5)\, d\theta$

Monday, February 25, 2019 12:47 PM

Example 6

Evaluate $\int e^{-9x}\,dx$

Example 7 – Additional Step Necessary

Evaluate $\int x\sqrt{5x+1}\,dx$

$u = 5x+1 \longrightarrow u-1 = 5x \qquad x = \frac{u-1}{5}$

$du = 5\,dx$

$\frac{1}{5}\,du = dx$

$\frac{1}{5}\int x\sqrt{u}\,du$

$\frac{1}{5}\int \left(\frac{u-1}{5}\right)\cdot u^{1/2}\,du$

$\frac{1}{25}\int u^{3/2} - u^{1/2}\,du$

$\frac{1}{25}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right)$

$\frac{1}{25}\left(\frac{2}{5}(5x+1)^{5/2} - \frac{2}{3}(5x+1)^{3/2}\right)$

Monday, February 25, 2019 12:47 PM

Example 8

Evaluate $\int_0^2 x^2\sqrt{x^3+1}\,dx$

Monday, February 25, 2019 12:47 PM

Example 9

Evaluate $\int_0^{\pi/4} \tan^3 x \sec^2 x \, dx$

$u = \tan 0$

$u = \tan \pi/4 = 1$

$u = \tan x$

$du = \sec^2 x \, dx$

$\int u^3 du$

$\frac{1}{4} u^4 \Big|_0^1 = \frac{1}{4} \cdot 1^4 - \frac{1}{4} \cdot 0^4 = \frac{1}{4}$

Monday, February 25, 2019 12:47 PM

Example 10

Calculate the area under the graph of $y = \dfrac{x}{x^2+1}$ over [1, 3]

$$\int_1^3 \frac{x}{x^2+1}\,dx$$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\frac{1}{2}\int \frac{du}{u}$$

$$\frac{1}{2}\ln|u|\Big|_2^{10}$$

$$= \frac{1}{2}(\ln 10 - \ln 2)$$

$$= \frac{1}{2}\ln 5$$

350

46, 47, 53, 56, 60