

## 5.6 examples

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### 5.6 examples

#### Calculus AB: U-Substitution (5.6)

$$u = \sin(x^3) \quad \times$$

$$u = 3x^2 \quad \times$$

#### Example 1

Evaluate  $\int 3x^2 \sin(x^3) dx$

$$u = x^3$$

$$\frac{du}{dx} = 3x^2$$

$$du = 3x^2 dx \quad \text{"differential form"}$$

$$\int \sin u \, du \quad \rightarrow \quad -\cos x^3 + C$$

$-\cos u$

Example 2Evaluate  $\int 2x(x^2 + 9)^5 dx$ 

$$u = x^2 + 9$$

$$du = 2x dx$$

$$\int u^5 du$$

$$\frac{u^6}{6}$$

$$\frac{(x^2 + 9)^6}{6} + C$$

Example 3 – Integral of  $\tan \theta$

Evaluate  $\int \tan \theta \, d\theta = \int \frac{\sin \theta}{\cos \theta} \, d\theta$

$u = \sin \theta$

$du = \cos \theta \, d\theta$

X

$u = \cos \theta$

$du = -\sin \theta \, d\theta$

$-du = \sin \theta \, d\theta$

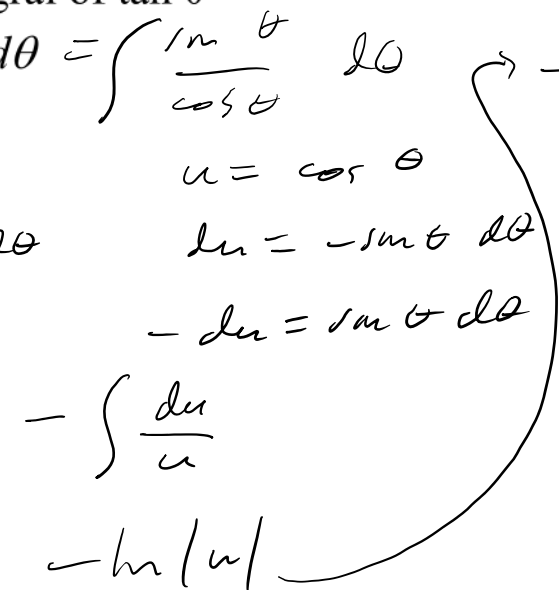
$-\int \frac{du}{u}$

$-\ln |u|$

$-\ln |\cos \theta| + C$

$\ln |\cos \theta| + C$

$\ln |\sec \theta| + C$



Example 4 – Multiplying du by a Constant

Evaluate  $\int \frac{x^2 + 2x}{(x^3 + 3x^2 + 9)^4} dx$

$$u = x^3 + 3x^2 + 9$$

$$du = 3x^2 + 6x dx$$

$$\frac{1}{3} du = x^2 + 2x dx$$

$$\frac{1}{3} \int \frac{du}{u^4}$$

$$\frac{1}{3} \int u^{-4} du$$

$$\frac{1}{3} \cdot \frac{u^{-3}}{-3}$$

$$-\frac{1}{9} \cdot \frac{1}{(x^3 + 3x^2 + 9)^3} + C$$

$$341: 2, 10, 12-14$$

$$350: 33 - 36$$

Example 5

Evaluate  $\int \sin(7\theta + 5) d\theta$

Example 6Evaluate  $\int e^{-9x} dx$ 

$$u = -9x$$

$$du = -9 dx$$

$$-\frac{1}{9} du = dx$$

$$-\frac{1}{9} \int e^u du$$

$$-\frac{1}{9} e^u$$

$$-\frac{1}{9} e^{-9x} + C$$



Example 7 – Additional Step NecessaryEvaluate  $\int x\sqrt{5x+1} dx$ 

$$u = 5x + 1 \longrightarrow u - 1 = 5x$$

$$du = 5 dx$$

$$\frac{u-1}{5} = \textcircled{x}$$

$$\frac{1}{5} du = dx$$

$$\frac{1}{5} \int \textcircled{x} \cdot u^{1/2} du$$

$$\frac{1}{5} \int \frac{u-1}{5} \cdot u^{1/2} du$$

$$\frac{1}{25} \int u^{3/2} - u^{1/2} du$$

$$\frac{1}{25} \left( \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right)$$

$$\frac{2}{125} u^{5/2} - \frac{2}{75} u^{3/2}$$

$$\frac{2}{125} (5x+1)^{5/2} - \frac{2}{75} (5x+1)^{3/2} + C$$

Example 8

Evaluate  $\int_0^2 x^2 \sqrt{x^3 + 1} dx$



Example 9

Evaluate  $\int_0^{\pi/4} \tan^3 x \sec^2 x \, dx$

$$u = \tan x$$

$$du = \sec^2 x \, dx$$

$$\int u^3 \, du$$

$$\frac{1}{4} u^4$$

$$\frac{1}{4} \tan^4 x \Big|_0^{\pi/4}$$

$$\frac{1}{4} (\tan^4 \frac{\pi}{4} - \tan^4 0)$$

$$= \frac{1}{4} (1 - 0)$$

$$= \frac{1}{4}$$

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$$\frac{1}{4} u^4 \Big|_0^1 = \frac{1}{4} (1^4 - 0^4)$$

$$= \frac{1}{4}$$

349: 40, 46, 51, 52, 53, 55, 56

**Example 10**

Calculate the area under the graph of  $y = \frac{x}{x^2 + 1}$  over  $[1, 3]$