5.8 examples

Calculus AB – 5.8 (Exponential Growth and Decay)

Example 1

$P(t) = P_0 e^{kt}$

When the E-coli bacteria (found in the human intestine) is grown in the laboratory, it increases exponentially with a growth constant of k = 0.41 $(\text{hours})^{-1}$. Assume that 1,000 bacteria are present at time t = 0.

a. Find the formula for the number of bacteria P(t) at time t.

$P(t) = 1000e^{0.41t}$

b. How large is the population after 5 hours?

c. When will the population reach 10,000?

$$1000e^{0.41t} = 10,000$$

$$\ln(e^{0.41t} = 10)$$

$$0.41t = \ln 10$$

$$t = \frac{\ln 10}{0.41}$$

Wednesday, March 6, 2019 10:24 PM

Example 2

$y' = ky \longrightarrow y = y_0 e^{kt}$

Find all solutions of $y' = 3y$. Which particular solution satisfies y(0) = 9?

$y = 9e^{3x}$

Example 3 – Modeling Penicillin

Pharmacologists have shown that the rate at which penicillin leaves a person's bloodstream is proportional to the amount of penicillin present.

a. Express this statement in a differential equation.

$$\frac{dP}{dt} = kP$$

Wednesday, March 6, 2019 10:24 PM

$\frac{dp}{dt} = kp \longrightarrow P = P_0 e^{kt}$

b. Find the decay constant if 50 mg of penicillin remain in the bloodstream 7 hours after an initial injection of 450 mg.

$50 = 450 e^{-7k}$

$\frac{1}{9} = e^{-7k}$

$\ln \frac{1}{9} = -7k$

$k = \frac{\ln 1/9}{-7} = 0.314$

c. Under the hypothesis of (b), at what time was 200 mg of penicillin present?

$200 = 450 e^{-0.314t}$

$\frac{4}{9} = e^{-0.314t}$

$\ln \frac{4}{9} = -0.314t$

$t = \frac{\ln 4/9}{-0.314}$

$= 2.582$ hours

Example 4 – Computing Doubling Time from k

Some studies have suggested that from 1955 to 1970, the number of bachelor's degrees in physics awarded per year by U.S. universities grew exponentially, with growth constant k = 0.1 (approximately 2,500 degrees awarded in 1955). If this was true,

a. What was the doubling time? $= \frac{\ln 2}{0.1} = 6.931$ yrs.

b. How long would it take for the number of degrees awarded per year to increase 8-fold, assuming that exponential growth continued?

≈ 20.8 yrs

Example 5 – Calculating k from the Doubling Time
One of the world's smallest flowering plants, Wolffia globosa, has a doubling time of approximately 30 hours. Find the growth constant k and determine the initial population if the population grew to 1,000 after 48 hours.

Example 6

The isotope Radon-222 has a half life of 3.825 days. Find the decay constant and determine how long it will take for 80% of the isotope to decay.

Example 7 – Lascaux Cave Paintings

$k = 0.000121$

A remarkable gallery of prehistoric paintings of animals was discovered in the Lascaux cave in Dordogne, France in 1940. Scientists determined that a charcoal sample taken from the cave had a C^{14} to C^{12} ratio equal to 15% of that found in the atmosphere. What did they estimate the age of the paintings to be?

$$0.15 = 1e^{-0.000121t}$$

$$\ln 0.15 = -0.000121t$$

$$t = \frac{\ln 0.15}{-0.000121} = 15,678 \text{ years}$$

366: 7-9, 11-13, 19, 20

Example 8

A principal $P_0 = \$10,000$ is deposited into an account paying 6% interest. Find the balance after 3 years if interest is compounded quarterly and if interest is compounded continuously.

$P(t) = P_0 e^{kt}$ is based on

$$\frac{dy}{dt} \sim y$$

$$\boxed{\frac{dy}{dt} = ky} \xrightarrow{\text{solve}} y = y_0 e^{kt}$$

$$\frac{dy}{dt} = \underbrace{y_0 e^{kt}}_{y} \cdot k = ky$$