Monday, June 10, 2019 9:31 AM

$$\int_0^1 \cos x^2 \, dx$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} \cdots$$

$$\cos x^2 = 1 - \frac{x^4}{2} + \frac{x^8}{4!} \cdots$$

$$\int_0^1 1 - \frac{x^4}{2} + \frac{x^8}{24} \, dx$$

$$= x - \frac{x^5}{10} + \frac{x^9}{9(24)} \Big|_0^1$$

$$= 1 - \frac{1}{10} + \frac{1}{216}$$

Find Taylor poly. for $f(x) = x^2 \cos x$

$= x^2\left(1 - \frac{x^2}{2} + \frac{x^4}{24} \cdots\right)$

$x^2 - \frac{x^4}{2} + \frac{x^6}{24} - \cdots$

find Taylor poly. for

$f(x) = e^x \sin x$

$$(1 + x)\left(x - \frac{x^3}{6}\right)$$

$$x - \frac{x^3}{6} + x^2 - \frac{x^4}{6}$$

$$x + x^2 - \frac{x^3}{6} - \frac{x^4}{6}$$

$$\sum_{n=1}^{\infty} \frac{2^n x^n}{n}$$ where does it converge?

$$= \frac{2x}{1} + \frac{4x^2}{2} + \frac{8x^3}{3} + \cdots \qquad -\tfrac{1}{2} \le x$$

$$\sqrt[n]{\frac{2^n x^n}{n}} = |2x| < 1 \qquad \left[-\tfrac{1}{2}, \tfrac{1}{2}\right)$$

check endpoints, $x = -\frac{1}{2}$

$$-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} \cdots \qquad C.$$

$x = 1/2$;

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \qquad D.$$

$$\sum_{n=1}^{\infty} \frac{x^n}{(n!)^2}$$ Use ratio test to find interval of convergence

$$\frac{\dfrac{x^{n+1}}{(n+1)!\,(n+1)!}}{\dfrac{x^n}{n!\,n!}} = \frac{x^{n+1}\,n!\,n!}{x^n\,(n+1)!\,(n+1)!} = \frac{x}{(n+1)(n+1)}$$

let $n \to \infty$ $= 0$

converges on $(-\infty, \infty)$

Taylor Series Error Bound

What is the max error if you use 3 terms of $g(x) = \sin x$ (center $= 0$) to estimate $\sin 0.5$?

(use alternating series error approach.)

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{0x^6}{6!}$$

$$\text{error} < \frac{x^7}{7!} \qquad \text{error} < \frac{0.5^7}{7!}$$