

$$\text{Given } f(x) = \frac{5}{1-3x} = 5 + 15x + 45x^2 + \dots$$

Find the associated Taylor polynomial.

Find the antiderivative of $f(x)$ and its associated Taylor polynomial.

$$\text{Find } \int f(x) dx. \quad \text{Use } u = 1-3x \quad du = -3 dx$$

$$-\frac{1}{3} du = dx \quad -\frac{1}{3} \int \frac{5}{u} du = -\frac{5}{3} \ln |u|$$

$$-\frac{5}{3} \ln |1-3x| = 5x + \frac{15}{2}x^2 + 15x^3 + \dots$$

$$\sum_{n=1}^{\infty} \frac{x^n}{n 2^n}$$

Find interval of convergence.

$$\sqrt[n]{n} \rightarrow 1$$

$$\sqrt[n]{\frac{x^n}{2^n}} = \frac{|x|}{2} < 1$$
$$[-2, 2)$$

Check endpoints $x = -2$

$$\frac{-2}{1 \cdot 2} + \frac{4}{2 \cdot 4} - \frac{8}{3 \cdot 8} + \dots \quad C$$

$x = 2$:

$$\frac{2}{1 \cdot 2} + \frac{4}{2 \cdot 4} + \frac{8}{3 \cdot 8} + \dots \quad D$$

Find a Taylor polynomial for

$$g(x) = x^3 \sin 2x$$

$$x^3 \left(2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} \right)$$

$$2x^4 - \frac{8x^6}{3!} + \frac{32x^8}{5!} + \dots$$

Use a Taylor polynomial with 3 terms to approximate

$$\int_0^1 \cos x^3 dx$$

$$\int_0^1 \left(1 - \frac{(x^3)^2}{2!} + \frac{(x^3)^4}{4!} \right) dx = \int_0^1 \left(1 - \frac{x^6}{2!} + \frac{x^{12}}{4!} \right) dx$$

$$= x - \frac{x^7}{7} + \frac{x^{13}}{13(24)} \Big|_0^1 = 1 - \frac{1}{7} + \frac{1}{13(24)}$$