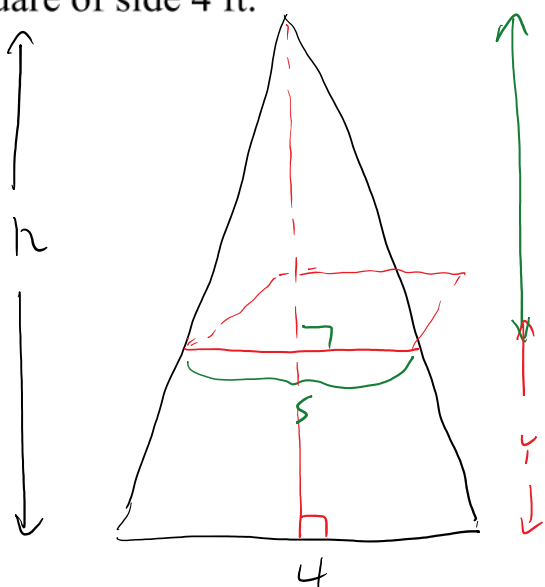




Calculus AB: Setting up Integrals (Volume, Average Value) Section 6.2

Example 1 – Volume of a Pyramid: Horizontal Cross Section

Calculate the volume V of a pyramid of height 12 ft whose base is a square of side 4 ft.



$$12 - y = \frac{12}{4} = \frac{12 - y}{s}$$

$$12s = 48 - 4y$$

$$s = 4 - \frac{1}{3}y$$

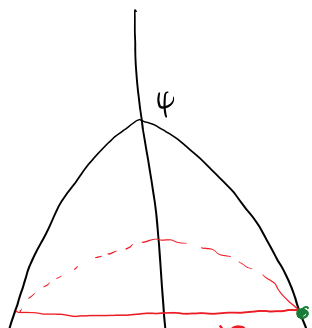
cross-sectional area =

$$s^2 = \left(4 - \frac{1}{3}y\right)^2 = A(y)$$

$$\int_0^{12} \left(4 - \frac{1}{3}y\right)^2 dy$$

Example 2

The base of a solid is the region between the x -axis and the inverted parabola $y = 4 - x^2$. The vertical cross-sections of the solid perpendicular to the y -axis are semi-circles. Compute the volume of the solid.



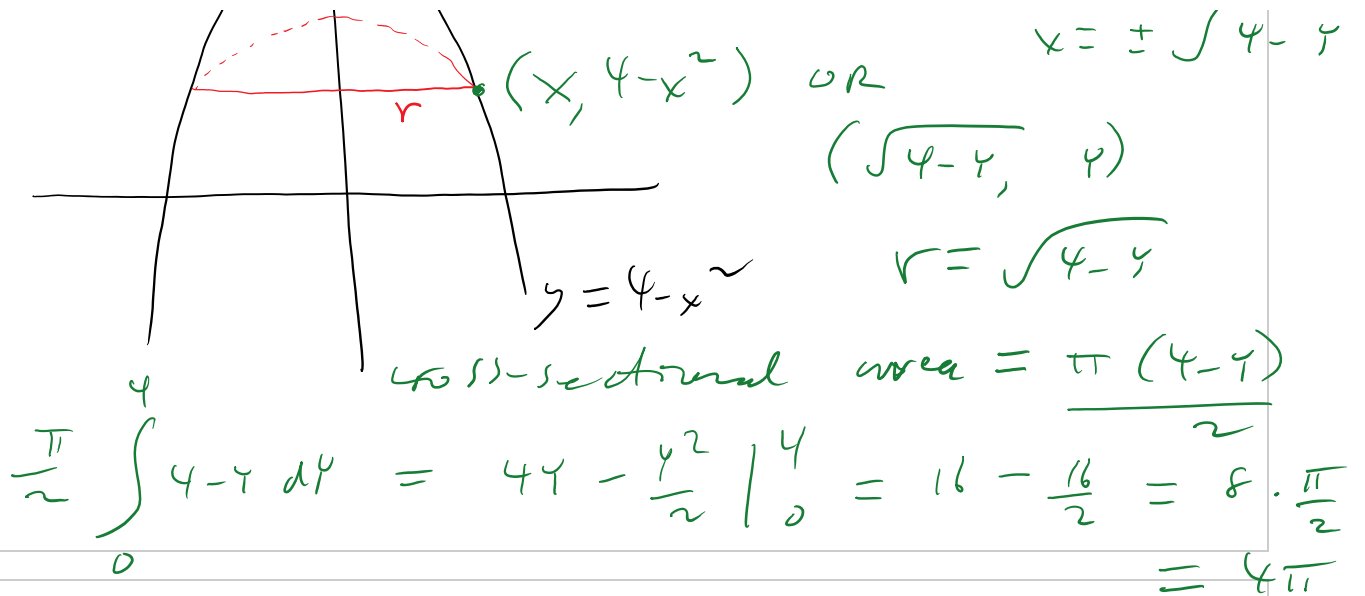
$$y = 4 - x^2$$

$$4 - y = -x^2$$

$$x^2 = 4 - y$$

$$x = \pm \sqrt{4 - y}$$

$$(x, 4 - x^2) \text{ OR}$$



Example 3 – Volume of a Sphere: Vertical Cross-Sections

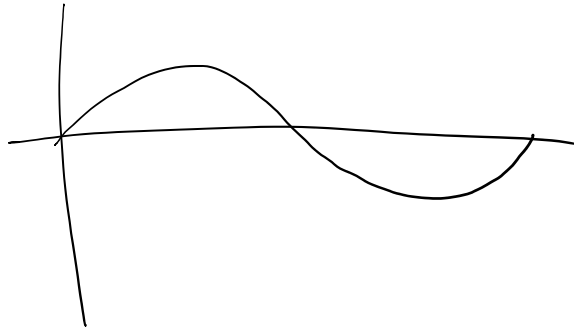
Compute the volume of a sphere of radius R as an integral of cross-sectional area.

Example 7

Find the average value of $f(x) = \sin x$ on

(a) $[0, \pi]$ $\frac{1}{\pi} \int_0^{\pi} \sin x \, dx = \frac{2}{\pi}$

(b) $[0, 2\pi]$



avg = 0

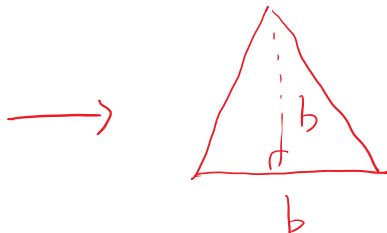
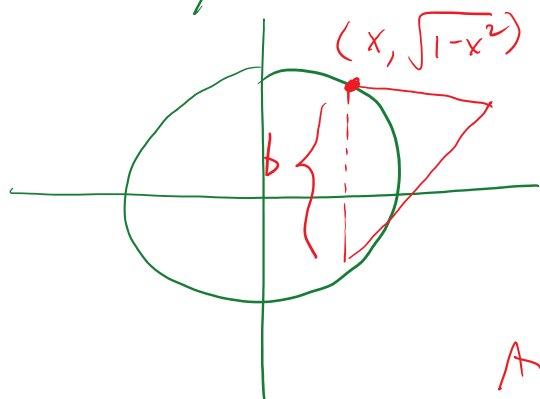
Example 8

A frog hops straight up from the ground with an initial velocity of $v_0 = 16$ ft/s. Use Galileo's formula for the height $h(t) = v_0 t - 16t^2$ to find the frog's average speed.

$$\text{speed} = |v| \quad h(t) = 16t - 16t^2$$
$$v(t) = 16 - 32t$$
$$\text{speed} = |16 - 32t|$$
$$\frac{1}{T} \int_0^T |16 - 32t| dt$$

$$\begin{array}{r} 392 \\ \hline 49, 51, 53, \\ 54 \end{array}$$

⑨ The base is the unit circle $x^2 + y^2 = 1$. The cross-sections \perp to the x -axis are triangles whose height and base are equal.



$$\begin{aligned} x^2 + y^2 &= 1 \\ y^2 &= 1 - x^2 \\ y &= \pm \sqrt{1 - x^2} \end{aligned}$$

$$\text{Area} = 2(1 - x^2)$$

$$\text{Volume} = 2 \int_{-1}^1 (1 - x^2) dx = 2 \left[x - \frac{x^3}{3} \Big|_{-1}^1 \right]$$

$$= 2 \left[1 - \frac{1}{3} - \left(-1 - \frac{(-1)^3}{3} \right) \right] = 2 \left(\frac{2}{3} - \left(-\frac{2}{3} \right) \right)$$

$$= \frac{8}{3}$$

390: 2, 4A, 4B, 11