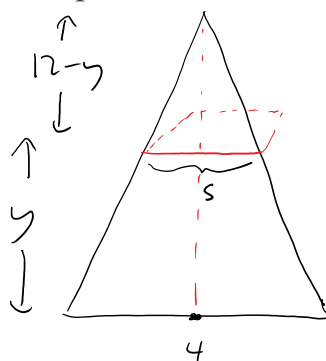




## Calculus AB: Setting up Integrals (Volume, Average Value) Section 6.2

### Example 1 – Volume of a Pyramid: Horizontal Cross Section

Calculate the volume  $V$  of a pyramid of height 12 ft whose base is a square of side 4 ft.



$$\frac{12}{4} = \frac{12-y}{s}$$

$$12s = 4(12-y)$$

$$s = \frac{1}{3}(12-y)$$

$$A(y) = s^2 = \frac{1}{9}(12-y)^2$$

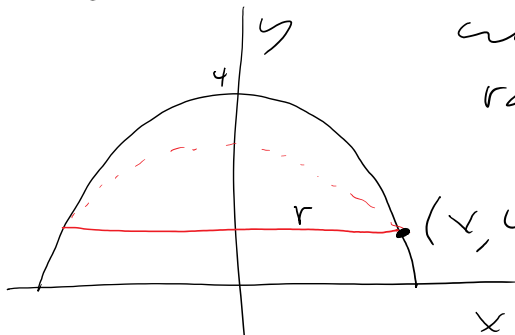
$$\frac{1}{9} \int_0^{12} (12-y)^2 dy = 64$$

$s(0) = 4$   
 $s(12) = 0$

Example 2

$$x = \sqrt{4-y}$$

The base of a solid is the region between the x-axis and the inverted parabola  $y = 4 - x^2$ . The vertical cross-sections of the solid perpendicular to the y-axis are semi-circles. Compute the volume of the solid.



cross-sectional area  
radius  $r = \sqrt{4-y}$

$$\text{Area} = \frac{\pi (4-y)}{2}$$

$$\begin{aligned} \text{Volume} &= \frac{\pi}{2} \int_0^4 (4-y) dy = 4y - \frac{1}{2}y^2 \Big|_0^4 \\ &= 16 - 8 = \frac{8 \cdot \pi}{2} = 4\pi \end{aligned}$$

Example 3 – Volume of a Sphere: Vertical Cross-Sections

Compute the volume of a sphere of radius  $R$  as an integral of cross-sectional area.

Example 7

Find the average value of  $f(x) = \sin x$  on

$$\frac{1}{b-a} \int_a^b f(x) dx$$

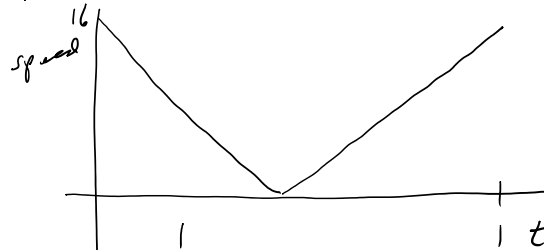
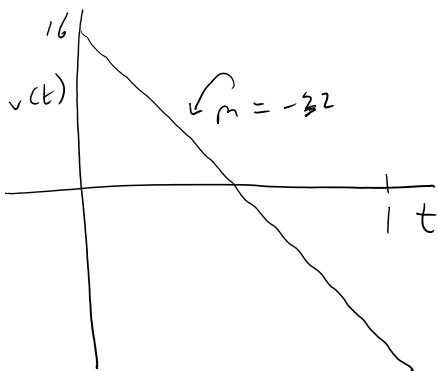
(a)  $[0, \pi]$   $\frac{1}{\pi-0} \int_0^{\pi} \sin x dx$

$$= \frac{1}{\pi} \cdot (-\cos x) \Big|_0^{\pi} = \frac{1}{\pi} (-\cos \pi - (-\cos 0))$$
$$= \frac{2}{\pi} \checkmark$$

(b)  $[0, 2\pi]$

### Example 8

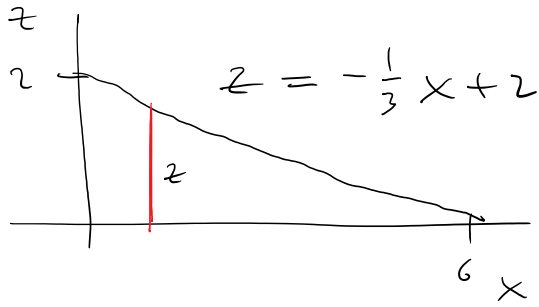
A frog hops straight up from the ground with an initial velocity of  $v_0 = 16$  ft/s. Use Galileo's formula for the height  $h(t) = v_0 t - 16t^2$  to find the frog's average speed.  $v(t) = 16 - 32t$



$$\frac{1}{1} \cdot \int_0^1 |16 - 32t| dt = 8$$

$$\frac{1}{\frac{1}{2}} \int_0^{\frac{1}{2}} 16 - 32t dt = 8$$

Cross-sections  $\perp$  to  $x$ -axis  
(rectangles)



$$\int_0^6 \frac{4x}{3} dx = 24$$

$$\int_0^6 \left(-\frac{1}{3}x + 2\right) 4 dx$$

$$4 \int_0^6 \left(-\frac{1}{3}x + 2\right) dx = 4 \left(-\frac{1}{6}x^2 + 2x\right) \Big|_0^6$$

$$= 4 \left[-\frac{1}{6}(6)^2 + 2(6)\right] = 4(-6 + 12) = 24$$

$$\textcircled{5} \int_0^4 6 dy = 24$$

390: 8, 10, 11