6.3 examples (1)

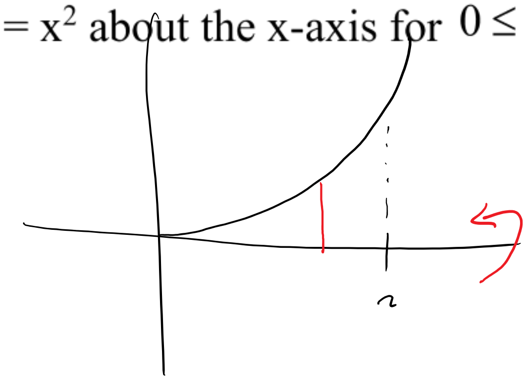
Monday, January 7, 2019 6:50 AM

6.3 examples (1)

Calculus AB: Volumes of Revolution (section 6.3)

<u>Example 1</u>

Calculate the volume V of the solid obtained by rotating the region under $y = x^2$ about the x-axis for $0 \le x \le 2$.



$f(x) = x^2$

$f(x)^2 = x^4$

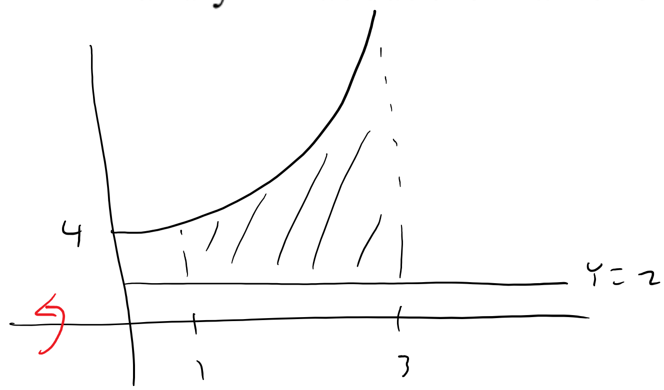
$\pi \int_0^2 x^4 \, dx = \pi \cdot \frac{x^5}{5} \Big|_0^2$

$= \pi \cdot \frac{32}{5}$

$\frac{39d}{7, 9, 11, 12}$

Example 2 – Rotating the Area Between Two Curves

Find the volume V of the solid obtained by rotating the region between $y = x^2 + 4$ and $y = 2$ about the x-axis for $1 \le x \le 3$.



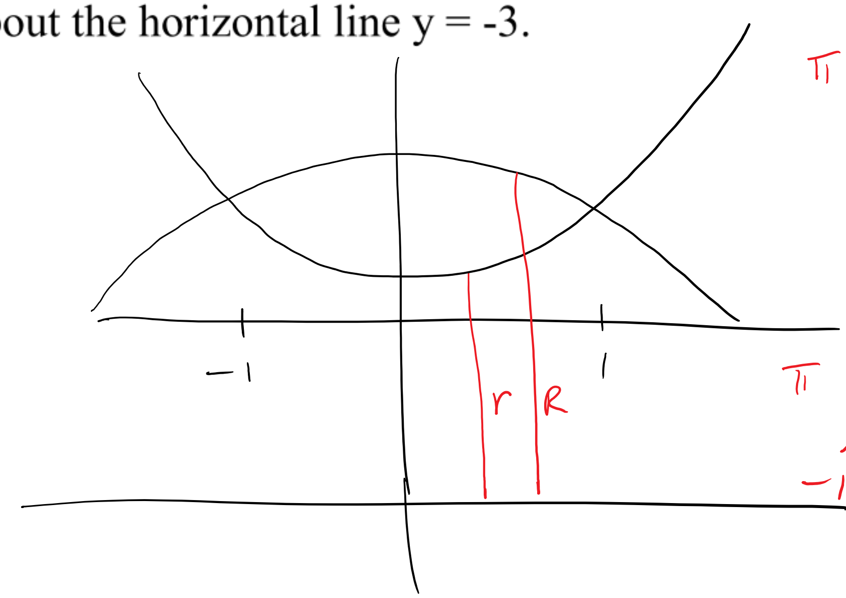
$$\pi \int f(x)^2 - g(x)^2 \, dx$$

$$\pi \int_1^3 (x^2+4)^2 - 2^2 \, dx$$

141.733π

<u>Example 3</u> – Revolving About a Horizontal Axis

Find the volume V of the “wedding band” in Figure 7(C), obtained by rotating the region between the graphs of $f(x) = x^2 + 2$ and $g(x) = 4 - x^2$ about the horizontal line $y = -3$.



$y = -3$

$\pi \int R^2 - r^2 \, dx$

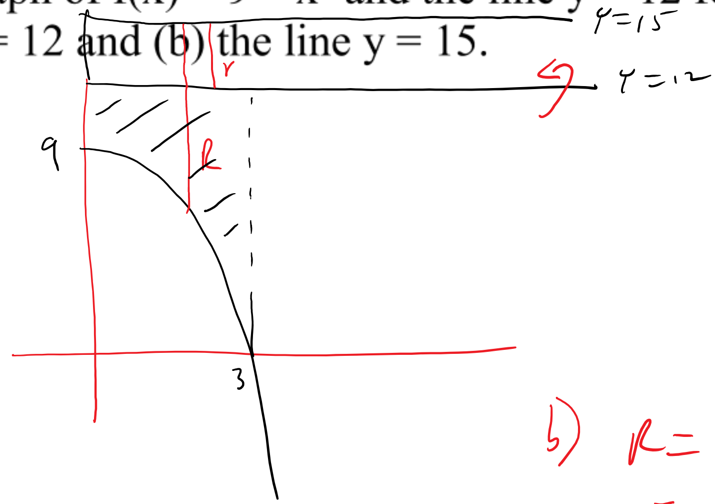
$r = x^2 + 2 + 3$

$R = 4 - x^2 + 3$

$\pi \int_{-1}^{1} (7 - x^2)^2 - (x^2 + 5)^2 \, dx$

<u>Example 4</u>

Find the volume V of the solid obtained by rotating the region between the graph of $f(x) = 9 - x^2$ and the line $y = 12$ for $0 \le x \le 3$ about (a) the line $y = 12$ and (b) the line $y = 15$.



a) disk method

$R = 12 - (9 - x^2)$

$= 3 + x^2$

$\pi \int_0^3 (3+x^2)^2 \, dx$

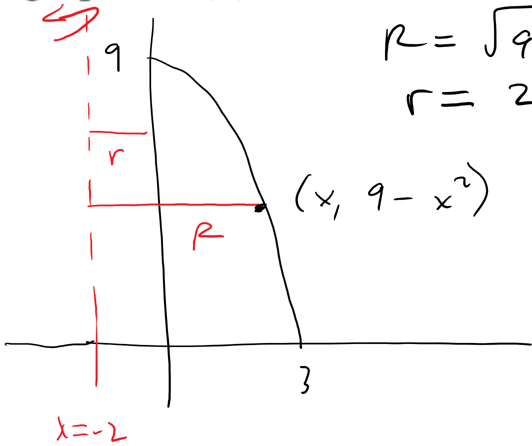
b) $R = 15 - (9 - x^2) = 6 + x^2$

$r = 3$

$\pi \int_0^3 (6+x^2)^2 - 3^2 \, dx$

Example 5 – Revolving About a Vertical Axis

Find the volume of the solid obtained by revolving the region under the graph of $f(x) = 9 - x^2$ for $0 \le x \le 3$ about the vertical axis $x = -2$.



$R = \sqrt{9-y} + 2$

$r = 2$

$(x, 9 - x^2)$ or $(\sqrt{9-y}, y)$

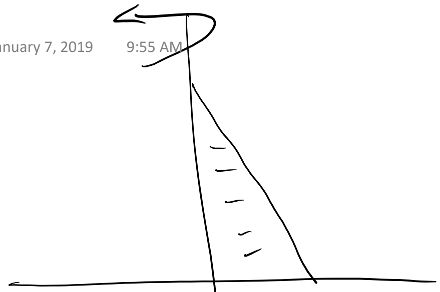
$y = 9 - x^2$

$x^2 = 9 - y$

$x = \sqrt{9-y}$

$$\pi \int_0^9 \left(\sqrt{9-y} + 2\right)^2 - 2^2 \, dy$$

399: 23–28



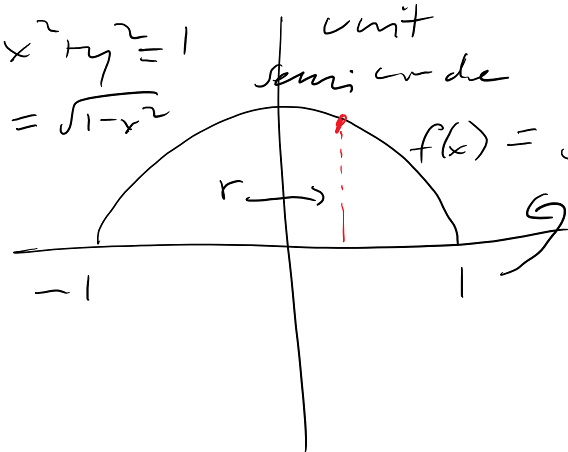
$\longrightarrow$ CONE

$x^2+y^2=1$

$y=\sqrt{1-x^2}$

unit semicircle

DISK METHOD



$f(x)=\sqrt{1-x^2}$

$\longrightarrow$ SPHERE

cross sectional area $= \pi r^2 = \pi f(x)^2$

$= \pi(1-x^2)$

volume $= \pi \int_{-1}^{1} 1-x^2\,dx = \pi\left(x-\frac{1}{3}x^3\right)\Big|_{-1}^{1}$

$\pi\left(1-\frac{1}{3}-\left(-1- -\frac{1}{3}\right)\right) = \pi\left(\frac{2}{3}-\left(-\frac{2}{3}\right)\right)$

$= \frac{4}{3}\pi$