7.2 examples

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7.2 examples

Calculus BC, section 7.2 - Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Example 1

Evaluate $\int x \cos x\,dx$

$u = x \qquad dv = \cos x\,dx$

$\frac{du}{dx} = 1 \qquad v = \sin x$

$du = dx$

$$\int u\,dv = x\sin x - \int \sin x\,dx$$

$$= x\sin x + \cos x + C$$

Logs
Inverse Trig
Polynomial
Exponential
Trig

The first one that appears is u

Example 2 - Good versus bad choices of u and v'

Evaluate $\int xe^x \, dx$ $\quad u = x \quad v = e^x$

$du = dx \quad dv = e^x$

$= xe^x - \int e^x dx = xe^x - e^x + C$

Example 3 - Taking v' = 1

Evaluate $\int_1^3 \ln x \, dx$ $u = \ln x$ $v = x$

$du = \frac{1}{x}$ $v' = dx$

$= x \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - x \Big|_1^3$

$= 3 \ln 3 - 3 - (1 \cdot \ln 1 - 1) =$

$3 \ln 3 - 2$

Example 4 – Integrating by parts more than once

Evaluate $\int x^2 \cos x \, dx$ $u = x^2$ $v = \sin x$

$du = 2x\,dx$ $v' = \cos x \, dx$

$= x^2 \sin x - 2\int x \sin x \, dx$

$\int x \sin x \, dx$: $u = x$ $v = -\cos x$

$du = dx$ $v' = \sin x \, dx$

$= -x\cos x - \int -\cos x \, dx = -x\cos x + \int \cos x$

$= -x\cos x + \sin x$

$x^2 \sin x - 2(-x\cos x + \sin x)$

$x^2 \sin x + 2x\cos x - 2\sin x + C$

Example 5 – Going in a circle?

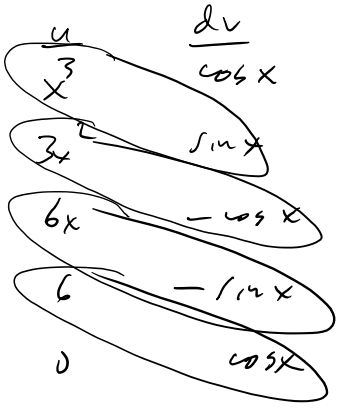
Evaluate $\int e^x \cos x \, dx$

Example 6 – Tabular Integration

Evaluate $\int x^3 \cos x \, dx$

$= x^3 \sin x + 3x^2 \cos x$

$- 6x \sin x - 6 \cos x + C$



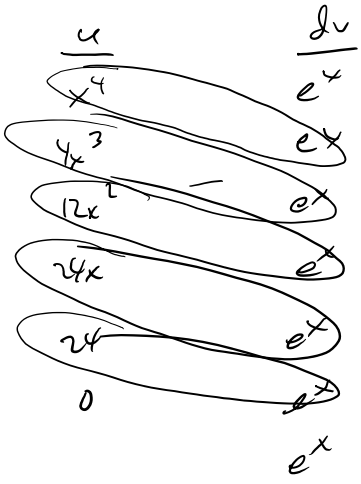
431

10, 13, 24, 28, 34, 41

Example 7 – Tabular Integration

Evaluate $\int x^4 e^x \, dx$ $= x^4e^x - 4x^3e^x + 12x^2e^x$

$-24xe^x + 24e^x$



Integration by Parts

$$(uv)' = u'v + uv'$$

$$\int (uv)' = \int u'v + \int uv'$$

$$\boxed{uv - \int u'v = \int uv'}$$