7.3 examples

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7.3 examples

Calculus BC, sect. 7.3 – Trigonometric Integrals

Trigonometry formulas:

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $\sec^2 x = \tan^2 x + 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$ $\qquad$ $\sin^2 x = \frac{1-\cos 2x}{2}$

$\sin(m+n) = \sin m \cos n + \cos m \sin n$

$\sin(m-n) = \sin m \cos n - \cos m \sin n$

$\cos(m + n) = \cos m \cos n - \sin m \sin n$

$\cos(m - n) = \cos m \cos n + \sin m \sin n$

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Example 1 – Odd Power of sin x

Evaluate $\int \sin^3 x \, dx$

$\int \sin^2 x \sin x \, dx$

$\int (1 - \cos^2 x) \sin x \, dx$

$\int \sin x - \cos^2 x \sin x \, dx$

$u^2 \quad -du$

$-\cos x + \int u^2 \, du$

$u = \cos x$

$du = -\sin x \, dx$

$-\cos x + \frac{1}{3} u^3$

$-\cos x + \frac{1}{3} (\cos x)^3 + C$

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Example 2 – Odd Power of sin x or cos x

Evaluate $\int \sin^4 x \cos^5 x \, dx$

$\int u^4 \, du - 2\int u^6 \, du + \int u^8 \, du$

$\int \sin^4 x \cos^2 x \cos^2 x \cos x \, dx$

$\int \sin^4 x (1-\sin^2 x)(1-\sin^2 x) \cos x \, dx$

$\int \sin^4 (1 - 2\sin^2 x + \sin^4 x) \cos x \, dx$

$\int \sin^4 x \cos x - 2\sin^6 x \cos x + \sin^8 x \cos x \, dx$

$u = \sin x \qquad du = \cos x \, dx$

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Example 3

$\sin^2 x = \frac{1-\cos 2x}{2}$

Evaluate $\int \sin^4 x\, dx$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$\int \sin^2 x \sin^2 x\, dx$

$\int \frac{1-\cos 2x}{2} \cdot \frac{1-\cos 2x}{2}\, dx$

$\frac{1}{4}\int 1 - 2\cos 2x + \cos^2 2x\, dx$

$\frac{1}{4}\int 1 - 2\cos 2x + \frac{1+\cos 4x}{2}\, dx$

$\frac{1}{4}\int 1.5 - 2\cos 2x + \frac{\cos 4x}{2}\, dx$

$\frac{1}{4}\left[1.5x - \sin 2x + \frac{1}{8}\sin 4x + C\right)$

$\frac{1}{4}\left(\frac{3}{2}\cdot\frac{\pi}{2} - 0 + \frac{1}{8}\cdot 0\right) = \frac{1}{4}\cdot\frac{3}{2}\cdot\frac{\pi}{2} + C = \frac{3\pi}{16} + C$

take 2

$\int \sin^4 x\, dx = \frac{\sin^3 x \cos x}{4} + \frac{3}{4}\int \sin^2 x\, dx$

$\int \sin^2 x\, dx = \frac{x}{2} - \frac{1}{2}\sin x \cos x$

$\int \sin^4 x\, dx = \frac{\sin^3 x \cos x}{4} + \frac{3}{4}\left(\frac{x}{2} - \frac{1}{2}\sin x \cos x\right)$

$\frac{3}{4}\cdot\frac{\pi/2}{2} = \frac{3\pi}{16} + C$

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$$\sec^2 x = 1 + \tan^2 x$$

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Example 4 – Even Powers of sin x and cos x

Evaluate $\int \sin^2 x \cos^2 x dx$ (problem modified)

Example 5 – Integral of the Tangent and Secant

Derive the formulas

$$\int \tan x \, dx = \ln | \sec x | + C$$

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$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

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Example 6 – Power of tan x

Evaluate $\int_0^{\pi/4} \tan^3 x \, dx$

Example 7 – Using Reduction Formulas and the Table of Integrals

Evaluate $\int \tan^2 x \sec^3 x \, dx$

Use $\sec^2 x = \tan^2 x + 1$ and

Reduction Formula (22): $\int \sec^m x \, dx = \frac{\tan x \sec^{m-2} x}{m-1} + \frac{m-2}{m-1} \int \sec^{m-2} x \, dx$

Example 8 – Integral of sin mx cos nx

Evaluate $\int_0^{\pi} \sin 4x \cos 3x \, dx$

a. Use the addition and subtraction formulas

b. Use Reduction Formula (26):

$$\int \sin mx \cos nx \, dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)} + C \quad (m \neq \pm n)$$