



### Calculus BC, section 7.6 – The Method of Partial Fractions

Example 1 – Finding the Constants  $A_1, A_2, \dots, A_n$

Evaluate  $\int \frac{dx}{x^2 - 7x + 10}$

$$\left[ \frac{1}{(x-5)(x-2)} = \frac{A}{x-5} + \frac{B}{x-2} \right] (x-5)(x-2)$$

$$1 = A(x-2) + B(x-5)$$

$$\text{Let } x=2: 1 = -3B \quad B = -1/3$$

$$\text{Let } x=5: 1 = A \cdot 3 \quad A = 1/3$$

$$\frac{1}{3} \int \frac{dx}{x-5} - \frac{1}{3} \int \frac{dx}{x-2} = \frac{1}{3} \ln|x-5| - \frac{1}{3} \ln|x-2| + C$$

Example 2

Evaluate  $\int \frac{x^2 + 2}{(x - 1)(2x - 8)(x + 2)} dx$

### Example 3 – Long Division Necessary

Evaluate  $\int \frac{x^3+1}{x^2-4} dx$  if numerator's order  $\geq$  denominator's order, divide

$$\begin{array}{r} x \\ x^2-4 \overline{) x^3 \phantom{+1} +1} \\ \underline{x^3 - 4x} \phantom{+1} \\ 4x + 1 \end{array} = x + \frac{4x+1}{x^2-4}$$

$$\frac{4x+1}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2}$$

$$4x+1 = A(x-2) + B(x+2)$$

$$x=2: 9 = 4B \quad B = 9/4$$

$$x=-2: -7 = -4A \quad A = 7/4$$

$$\frac{7}{4} \int \frac{dx}{x+2} + \frac{9}{4} \int \frac{dx}{x-2} = \frac{7}{4} \ln|x+2| + \frac{9}{4} \ln|x-2| + C$$

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Example 4 – Repeated Linear Factors

Evaluate  $\int \frac{3x - 9}{(x - 1)(x + 2)^2} dx$

Example 5 – Irreducible vs. Reducible Quadratic Factors

(a) Evaluate  $\int \frac{18}{(x+3)(x^2+9)} dx$

Evaluate  $\int \frac{18}{(x+3)(x^2-9)} dx$

Example 6 – Repeated Quadratic Factor

Evaluate  $\int \frac{4-x}{x(x^2+2)^2} dx$

$$\textcircled{5} \quad f(x) = \frac{x^3 + x + 1}{x - 2}$$

Find  $\int f(x) dx$