8.1 examples

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Calculus AB – Arc Length (section 8.1) $\int_a^b \sqrt{1 + f'(x)^2}\,dx$

Example 1

Calculate the arc length of the graph of $f(x) = \frac{1}{12}x^3 + x^{-1}$ over [1, 3].

$f'(x) = \frac{1}{4}x^2 - x^{-2}$

$$\int_1^3 \sqrt{1 + \left(\frac{1}{4}x^2 - x^{-2}\right)^2}\,dx$$

$\frac{485}{6} - 10, 11, 15$ use calculator

Example 2 – Arc Length as a Function of the Upper Limit

cosh x = "hyperbolic cosine of x" (pronounced "cosh x")

sinh x = "hyperbolic sine of x" (pronounced "cinch x")

$\cosh x = \dfrac{e^x + e^{-x}}{2}$ $\quad$ $\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\dfrac{d}{dx}\cosh x = \sinh x$ $\quad$ $\dfrac{d}{dx}\sinh x = \cosh x$

$1 + \sinh^2 x = \cosh^2 x$

→ Find the arc length of y = cosh x over [0, a].

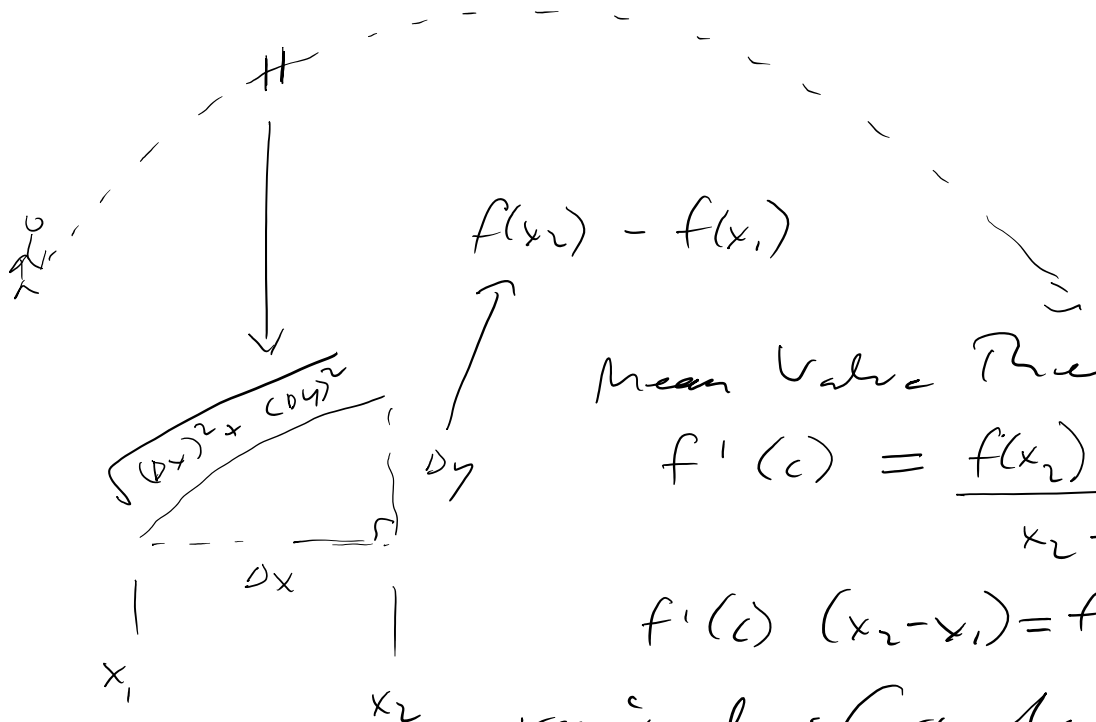
Then find the arc length over [0, 2].

Example 3 – When Arc Length Cannot Be Calculated Exactly

Express the arc length L of $y = \sin x$ over $[0, \pi]$ as an integral. Then approximate L using

(a) the Trapezoidal Rule with 6 trapezoids, and

(b) a computer algebra system



Mean Value Theorem

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$f'(c)\,(x_2 - x_1) = f(x_2) - f(x_1)$$

revised formula

$$\sqrt{(\Delta x)^2 + [f'(c)]^2 \cdot \Delta x^2}$$

$$\sqrt{(\Delta x)^2 \left(1 + f'(c)^2\right)}$$

$$\sqrt{1 + f'(c)^2} \cdot \Delta x$$

$$\lim_{\Delta x \to 0} \sqrt{1 + f'(x)^2} \cdot dx$$

$$\int_a^b \sqrt{1 + f'(x)^2}\, dx = \text{arc length from } x = a \text{ to } x = b$$