8.1 examples

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Calculus AB – Arc Length (section 8.1)

<u>Example 1</u>

$$\int \sqrt{1+f'(x)^2}\,dx$$

Calculate the arc length of the graph of f(x) = $\frac{1}{12}x^3 + x^{-1}$ over [1, 3].

$$f'(x) = \frac{1}{4}x^2 - x^{-2}$$

$$f'(x)^2 = \left(\frac{1}{4}x^2 - x^{-2}\right)^2$$

$$\int_1^3 \sqrt{1 + \left(\frac{1}{4}x^2 - x^{-2}\right)^2}\,dx = 2.\overline{83}$$

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Example 2 – Arc Length as a Function of the Upper Limit
cosh x = "hyperbolic cosine of x" (pronounced "cosh x")
sinh x = "hyperbolic sine of x" (pronounced "cinch x")

cosh x = $\dfrac{e^{x}+e^{-x}}{2}$ sinh x = $\dfrac{e^{x}-e^{-x}}{2}$

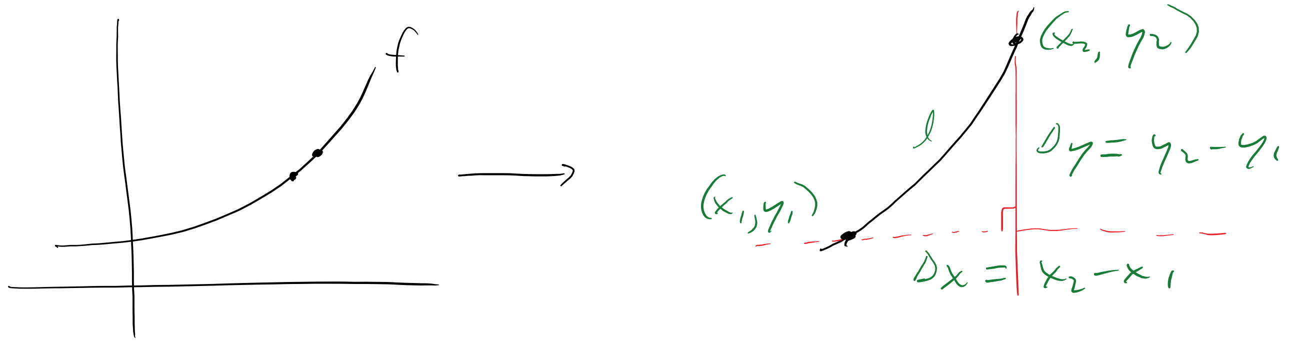
$\dfrac{d}{dx}\cosh x = \sinh x$ $\dfrac{d}{dx}\sinh x = \cosh x$

$1+\sinh^{2} x = \cosh^{2} x$

→ Find the arc length of y = cosh x over [0, a].
Then find the arc length over [0, 2].

Example 3 – When Arc Length Cannot Be Calculated Exactly
Express the arc length L of $y = \sin x$ over $[0, \pi]$ as an integral. Then approximate L using
(a) the Trapezoidal Rule with 6 trapezoids, and
(b) a computer algebra system

Derive the formula for path length



$$l^2 \approx \Delta x^2 + \Delta y^2$$

$$l \approx \sqrt{\Delta x^2 + \Delta y^2}$$

Mean Value Theorem

$\frac{\Delta y}{\Delta x} = y'(c)$ at some point c

$$\Delta y = y'(c)\,\Delta x$$

$$\Delta y^2 = (y'(c)\,\Delta x)^2$$

$$l \approx \sqrt{\Delta x^2 + (y'(c))^2 \Delta x^2}$$

$$\approx \sqrt{\Delta x^2 (1 + (y'(c))^2}$$

$$\approx \sqrt{1 + (y'(c))^2} \cdot \Delta x$$

$\lim_{\Delta x \to 0}$ This becomes $\sqrt{1 + y'(x)^2}\, dx = l$

$$\int_a^b \sqrt{1 + (y'(x))^2}\, dx = \text{arc length or path length}$$