8.4 examples

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Calculus BC, section 8.4 – Taylor Polynomials

Example 1 – Computing Taylor Polynomials Center = 3

Let f(x) = $\sqrt{x+1}$. Compute $T_n(x)$ at a = 3 for n = 0, 1, 2, 3 and 4.

Solution: First find the derivatives $f^{(j)}(3)$: $f(a) = f(3) = 2$

$$f'(x) = \frac{1}{2}(x+1)^{-1/2} = \frac{1}{2\sqrt{x+1}} = \frac{1}{4} = f'(a)$$

Then compute the coefficients $\frac{f^{(j)}(3)}{j!}$: $f''(x) = -\frac{1}{4}(x+1)^{-3/2} = -\frac{1}{4(x+1)^{3/2}}$

$$f''(3) = \frac{1}{4(4)^{3/2}} = -\frac{1}{32}$$

The first 4 terms are: $T_2(x) = 2 + \frac{\frac{1}{4}(x-3)}{1!} - \frac{\frac{1}{32}(x-3)^2}{2!}$

Example 2 – Find a General Formula for T_n

Find the Taylor polynomials $T_n(x)$ of $f(x) = \ln x$ at $a = 1$.
Find a formula for the nth Taylor polynomial.

Example 3 – Maclaurin Polynomials for f(x) = cos x

$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$

Find the Maclaurin polynomials of f(x) = cos x

Maclaurin polynomials = Taylor polynomials centered at a = 0

$\cos 0 = 1$

$(\cos x)' = -\sin x \qquad -\sin 0 = 0 \qquad \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}(-1)^n$

$(\cos x)'' = -\cos x \qquad -\cos 0 = -1$

$(\cos x)''' = \sin x \qquad \sin 0 = 0$

$(\cos x)^{(4)} = \cos x \qquad \cos 0 = 1$

What is interval of convergence?

$$\frac{x^{2n}}{(2n)!} \longrightarrow \frac{\frac{x^{2(n+1)}}{2(n+1)!}}{\frac{x^{2n}}{(2n)!}} \qquad \frac{x^{2n+2} \cdot (2n)!}{x^{2n} \cdot (2n+2)!}$$

$$\frac{x^2}{(2n+2)(2n+1)} \to 0$$

$\sum$ converges everywhere

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Example 4 – Using the Error Bound

Use the Error Bound to find a bound for the error $|T_3(1.2) - \ln 1.2|$, where $T_3(x)$ is the third Taylor polynomial for $f(x) = \ln x$ at $a = 1$. Check your result using a calculator.

Example 5 – Approximating With a Given Accuracy

Let $T_n(x)$ be the nth Maclaurin polynomial for $f(x) = \cos x$.
Find a value of n such that $|T_n(0.2) - \cos(0.2)| < 10^{-5}$

Use a calculator to verify that this value of n works.

Find the Maclaurin polynomials for $g(x) = e^x$. $a = 0$ $T_3(x)$

$$f(0) + \frac{f'(0)}{1!}(x-0) + \frac{f''(0)}{2!}(x-0)^2 + \frac{f'''(0)}{3!}(x-0)^3$$

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

On what x-interval does $T_n(x)$ converge to e^x?

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \frac{\frac{x^{n+1}}{(n+1)!}}{\frac{x^n}{n!}} \qquad \frac{x^{n+1} \cdot n!}{x^n \cdot (n+1)!}$$

$$\rightarrow \frac{x}{n+1} \rightarrow 0$$

$\sum$ converges for all x

Find $T_3(x)$ for e^x at center $a = 2$

$$e^2 + e^2(x-2) + \frac{e^2(x-2)^2}{2!} + \frac{e^2(x-2)^3}{3!}$$